Liquidity and Investment in General Equilibrium

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What is the SDF in an economy with incomplete markets and illiquid assets?

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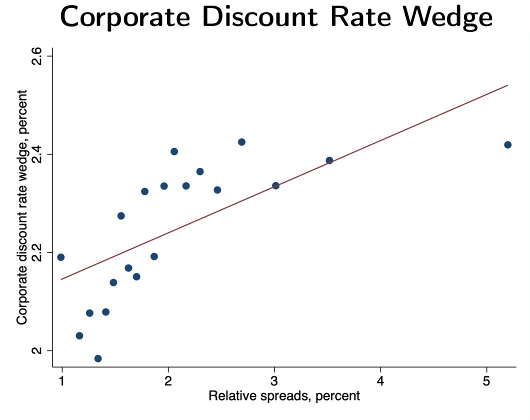

Discount rate wedge: Gap between discount rate and cost of capital (Gormsen Huber 2023). Relative spreads from CRSP.

Fact: Illiquid firms have higher SDF wedges

This paper:

- Rationalize this fact
- Implication for investment

Liquidity and investment in general equilibrium

Model

- Aiyagari production economy with liquid and illiquid assets in general equilibrium
- Firms take into account that ownership shares trade in frictional asset markets

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   - firms' SDF as if they have quasi-hyperbolic discounting
   - result from frictions in financial markets

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2. **Quantitative:** trading frictions & aggregate distortions
   - Trading frictions have adverse effects on capital without commitment
   - Counterfactual with commitment: trading frictions have little effect on capital

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2. **Quantitative:** trading frictions & aggregate distortions
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3. **Empirics:** rationalize facts on the cross-section of liquidity, investment, and SDF

Model

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Aiyagari production economy with liquid and illiquid assets

Households

- idiosyncratic labor risk h
- incomplete markets:
  - liquid bond b, borrowing limit $b \geq \underline{b}$
  - illiquid stock θ, transaction costs $\mathcal{T}$

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Firms

- DRS technology $y = \left(h^{\gamma} k^{1-\gamma}\right)^{\psi}$
- capital accumulation $k_{t+1} = i_t + (1-\delta)k_t \longleftarrow$ firms solve a **dynamic** problem

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Stationary equilibrium

- interest rate r, stock price q, and wage w such that markets clear:

$$\mathbb{E}[b] = 0 \qquad \mathbb{E}[\theta] = 1 \qquad \mathbb{E}[h] = H$$

Household problem

$$V(\theta, b, h) = \max_{c, b', \Delta^+, \Delta^-} u(c) + \beta \mathbb{E}\left[V\left(\theta', b', h'\right)\right]$$

subject to

$$c + b' + q\Delta^+ \leq wh + b(1+r) + d\theta + q\left(\Delta^- - \mathcal{T}\left(\Delta^-\right)\right)$$

$$\theta' = \theta + \Delta^+ - \Delta^-$$

$$\Delta^- \leq \theta \leftarrow \text{short-selling constraint}$$

$$b' \geq \underline{b} \leftarrow \text{borrowing constraint}$$

$$\mathcal{T}\left(\Delta^-\right) = \frac{\phi}{2}\left(\Delta^-\right)^2 \leftarrow \text{Transaction costs for sellers (e.g., Heaton Lucas 96)}$$

$$\Delta^+, \Delta^- \geq 0$$

Owners valuation

- Let $\tilde{q}(\theta, b, h)$ be owners's valuation in units of the consumption good

$$\tilde{q}(\theta, b, h) = \frac{V_\theta(\theta, b, h)}{u'(c)}$$

where V_θ is the marginal valuation of stocks.

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- Owners valuation is

$$\tilde{q}(\theta, b, h) = d + \left(1 - \phi\Delta^-(\theta, b, h)\right) q$$

- Buyers, $\Delta^- = 0$: agree the value of the firm is $\tilde{q}(\theta, b, h) = d + q$
- Sellers: Heterogeneous valuations, depend on marginal transaction cost $\phi\Delta^-$

$\rightarrow$ Disagreement among owners on the valuation of the firm

Firm's objective

Assumption 1: Firm maximizes owners' valuation weighted by ownership shares.

$$\int_{\theta,b,h} \theta \underbrace{\left[d + (1 - \phi\Delta^{-}(\theta, b, h))q\right]}_{\text{owners' valuation}} d\Gamma(\theta, b, h)$$

In spirit of Grossman Hart 1979 (paper also considers Dreze 1974 and DeMarzo 1993).

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Define $\bar{\Phi}$ as the weighted average marginal transaction cost

$$\bar{\Phi} \equiv \phi \int_{\theta,b,h} \theta\Delta^{-}(\theta, b, h) d\Gamma(\theta, b, h)$$

The firm maximizes

$$d + \left(1 - \bar{\Phi}\right) q$$

The frictionless case $\phi = 0$

- The firm's objective is to maximize $d + q$
- The price is equal to $q = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t d_t$
- Standard time-consistent problem
- Maximize the NPV of dividends, discounted at the risk-free rate

$\rightarrow$ deviations from *standard discounting* come from transaction costs, $\phi > 0$

Time Inconsistency in a Three-Period Model

Three-period model

Simplified model to show the time inconsistency problem

- Three periods: $t \in \{0, 1, 2\}$
- No income risk, two type of households with income $\{H, L, H\}$ and $\{L, H, L\}$
- No bonds

Three-period model: Euler equations & firm's value

Euler equations:

$$\left(1-\phi\Delta_0^{j-}\right)q_0=\beta\frac{u'\left(c_1^j\right)}{u'\left(c_0^j\right)}d_1+\beta\frac{u'\left(c_1^j\right)}{u'\left(c_0^j\right)}\left(1-\phi\Delta_1^{j-}\right)q_1$$

$$\left(1-\phi\Delta_1^{j-}\right)q_1=\beta\frac{u'\left(c_2^j\right)}{u'\left(c_1^j\right)}d_2$$

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$$\left(1 - \phi\Delta_1^{j-}\right) q_1 = \beta \frac{u'\left(c_2^j\right)}{u'\left(c_1^j\right)} d_2$$

Firm's value:

$$\sum_{j\in\{l,h\}} \frac{\theta_0^j}{2} \left[d_0 + (1 - \phi\Delta_0^{j-}) q_0\right]$$

$$\sum_j \frac{\theta_0^j}{2} \left[d_0 + \beta \frac{u'\left(c_1^j\right)}{u'\left(c_0^j\right)} d_1 + \beta^2 \frac{u'\left(c_2^j\right)}{u'\left(c_0^j\right)} d_2\right]$$

Time consistency in the three-period model

Problem in period 0

$$\max_{k_1,k_2\geq 0} \sum_j \frac{\theta_0^j}{2}\left[d_0 + \beta \frac{u'\left(c_1^j\right)}{u'\left(c_0^j\right)} d_1 + \beta^2 \frac{u'\left(c_2^j\right)}{u'\left(c_0^j\right)} d_2\right]$$

Problem in period 1

$$\max_{k_2\geq 0} \sum_j \frac{\theta_1^j}{2}\left[d_1 + \beta \frac{u'\left(c_2^j\right)}{u'\left(c_1^j\right)} d_2\right]$$

Time consistency in the three-period model

Problem in period 0

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The problem is **time consistent** iff the discounting between period 1 and 2 coincides

$$\underbrace{\frac{\sum_j \frac{\theta_0^j}{2}\beta^2\frac{u'(c_2^j)}{u'(c_0^j)}}{\sum_j \frac{\theta_0^j}{2}\beta\frac{u'(c_1^j)}{u'(c_0^j)}}}_{\substack{t=0\text{ discount between}\\ t=1\text{ and }t=2}} = \underbrace{\sum_j \frac{\theta_1^j}{2}\beta\frac{u'(c_2^j)}{u'(c_1^j)}}_{\substack{t=1\text{ discount between}\\ t=1\text{ and }t=2}}$$

Three-period model, frictionless case $\phi = 0$

The Euler equation implies equalization of marginal rates of substitution across agents:

$$\beta \frac{u'\left(c_{t+1}^j\right)}{u'\left(c_t^j\right)} = \frac{q_t}{d_{t+1} + q_{t+1}}$$

Hence

$$\underbrace{\frac{\sum_j \frac{\theta_0^j}{2} \beta^2 \frac{u'(c_2^j)}{u'(c_0^j)}}{\sum_j \frac{\theta_0^j}{2} \beta \frac{u'(c_1^j)}{u'(c_0^j)}}}_{\substack{t=0 \text{ discount between} \\ t=1 \text{ and } t=2}} =$$

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- The problem is time consistent when $\phi = 0$

Three-period model with trading frictions, $\phi > 0$

With transaction costs:

$$\frac{\sum_j \frac{\theta_0^j}{2} \beta^2 \frac{u'(c_2^j)}{u'(c_0^j)}}{\sum_j \frac{\theta_0^j}{2} \beta \frac{u'(c_1^j)}{u'(c_0^j)}} \neq \sum_j \frac{\theta_1^j}{2} \beta \frac{u'(c_2^j)}{u'(c_1^j)}$$

- The intertemporal marginal rates of substitution are **not** equalized across agents
- The problem is time inconsistent

Infinite-Horizon Model

Euler equation

$$(1-\phi\Delta_t^-)q_t = E_t\left[\beta\frac{u'(c_{t+1})}{u'(c_t)}\right](d_{t+1} + (1-\Phi_t)\, q_t) + \eta_t$$

where η_t is the Lagrange multiplier on $\Delta^- \leq \theta$ and

$$\Phi_t \equiv \mathbb{E}_t\left[\phi\Delta_{t+1}^-\right] + \phi\frac{\text{cov}_t\left(u'(c_{t+1}), \Delta_{t+1}^-\right)}{\mathbb{E}_t\left[u'(c_{t+1})\right]}$$

Φ captures liquidity frictions:

1. Expected marginal transaction costs: $\phi\Delta_{t+1}^- \rightarrow$ lower asset prices
2. Positive covariance if sell in bad times $\rightarrow$ further depress asset prices

The liquidity premium

- Focus on unconstrained buyers: $\Delta_t^- = 0$, $\Delta_t^+ > 0$, $b_{t+1} > \underline{b}$
- Bonds' Euler equation

$$\frac{1}{1+r_t} = E_t\left[\beta \frac{u'(c_{t+1})}{u'(c_t)}\right]$$

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- Bonds' Euler equation

$$\frac{1}{1+r_t} = E_t\left[\beta \frac{u'(c_{t+1})}{u'(c_t)}\right]$$

- Asset price:

$$q_t = \frac{d_{t+1} + (1-\Phi)\, q_{t+1}}{1+r}$$

- **Liquidity premium**
  - Define the yield of the stock as

$$1 + r^\theta \equiv \frac{d_{t+1} + q_{t+1}}{q_t}$$

  - The liquidity premium is $r^\theta - r = \Phi$

Assumption 2: The firm takes $\bar{\Phi}$ and Φ as given.

Firm's problem

$$V^F(k_t) = \max_{\{k_{t+s}\}_{s\geq 1}} d_t + (1 - \bar{\Phi})q_t$$

subject to

$$q_t = \frac{d_{t+1} + (1 - \Phi)\, q_{t+1}}{1 + r}$$

where $d_t = F(k_t, k_{t+1}) = zk_t^{\alpha} + (1 - \delta)k_t - k_{t+1}$

▷ static labor choice

Quasi-hyperbolic discounting and time consistency

Quasi-hyperbolic discounting

Proposition: *we can cast the firm's problem as if it has quasi-hyperbolic discounting*

$$V^F(k_t) = \max_{\{k_{t+s}\}_{s\geq 1}} F(k_t, k_{t+1}) + \tilde{\beta} \sum_{s=1}^{\infty} \tilde{\delta}^s F(k_{t+s}, k_{t+s+1})$$

where

- $\tilde{\delta} = \frac{1-\Phi}{1+r}$ exponential discounting with liquidity premium
- $\tilde{\beta} = \frac{1-\bar{\Phi}}{1-\Phi}$ time-inconsistency
- quasi-hyperbolic discounting iff $\Phi \neq \bar{\Phi}$
- present bias (i.e., $\tilde{\beta} < 1$) iff $\bar{\Phi} > \Phi$

▷ static labor choice

Time inconsistency & present bias

Proposition: *the difference $\Phi - \bar{\Phi}$ is equal to persistence and risk premium effects:*

$$\Phi - \bar{\Phi} = \underbrace{\frac{\phi}{2}\left(\tilde{\mathbb{E}}\left[\mathbb{E}_t\left[\Delta^-_{t+1}\right] \middle\| \text{buyer}\right] - \tilde{\mathbb{E}}\left[\mathbb{E}_t\left[\Delta^-_{t+1}\right]\right]\right)}_{\text{persistence effect}} + \underbrace{\frac{\phi}{2}\tilde{\mathbb{E}}\left[\frac{\text{cov}_t\left(u'\left(c_{t+1}\right), \Delta^-_{t+1}\right)}{\mathbb{E}_t\left[u'\left(c_{t+1}\right)\right]} \middle\| \text{buyer}\right]}_{\text{risk premium}}$$

$\tilde{\mathbb{E}}$ is the cross-sectional expectation, weighted by stock shares θ'

No transaction costs: If $\phi = 0$ then $\Phi = \bar{\Phi} = 0$, so $\tilde{\beta} = 1$, time consistent problem.

Intuition: persistence and risk premium

Persistence effect: $$\frac{\phi}{2}\left(\tilde{\mathbb{E}}\left[\mathbb{E}_t\left[\Delta^-_{t+1}\right]\|\,\text{buyer}\right]-\tilde{\mathbb{E}}\left[\mathbb{E}_t\left[\Delta^-_{t+1}\right]\right]\right)$$

- difference on average transaction costs for buyers and owners
- smaller for buyers than owners $\rightarrow$ negative term

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- difference on average transaction costs for buyers and owners
- smaller for buyers than owners $\rightarrow$ negative term

Risk premium: $$\tilde{\mathbb{E}}\left[\frac{\text{cov}_t\left(u'\left(c_{t+1}\right), \Delta_{t+1}^{-}\right)}{\mathbb{E}_t\left[u'\left(c_{t+1}\right)\right]}\middle\| \text{buyer}\right]$$

- if sell in bad times $\rightarrow$ positive covariance

- quantitatively the persistence effect dominates, so $\tilde{\beta} < 1$
- the problem is time inconsistent and the firm has present bias

Solution with and without commitment

Solution with and without commitment

With commitment

$$\max_{\{k_{t+s}\}_{s\geq 1}} F(k_t, k_{t+1}) + \tilde{\beta} \sum_{s=1}^{\infty} \tilde{\delta}^s F(k_{t+s}, k_{t+s+1})$$

Steady state

- SDF: $\tilde{\delta}$
- Capital

$$k^C = \left(\frac{(1-\gamma)\,\psi\tilde{\delta}}{1-\tilde{\delta}\,(1-\delta)} H^{\gamma\psi} \right)^{\frac{1}{1-(1-\gamma)\psi}}$$

Solution with and without commitment

With commitment

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Without commitment

- Markov perfect equilibrium

$$\max_{k'} F(k, k') + \tilde{\beta}\tilde{\delta} W(k')$$

$$W(k') = F(k', g(k')) + \tilde{\delta} W(g(k'))$$

Steady state

- SDF: $\tilde{\beta}\tilde{\delta}$
- Capital

$$k^N = \left(\frac{(1-\gamma)\,\psi\tilde{\beta}\tilde{\delta}}{1-\tilde{\beta}\tilde{\delta}\,(1-\delta)} H^{\gamma\psi} \right)^{\frac{1}{1-(1-\gamma)\psi}}$$

Incomplete markets, transaction costs, and commitment

1. **Complete markets**
   - $\beta(1 + r) = 1$, firms discount at rate $\frac{1}{1+r} = \beta$

2. **Aiyagari 94:** incomplete markets without transactions costs
   - $\tilde{\beta} = 1$, no problems of commitment
   - firms discount at rate $\frac{1}{1+r}$
   - GE: precautionary savings, $\beta(1 + r) < 1$, *more capital* than in complete markets

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3. **Transactions costs, with commitment**
   - firms discount at rate $\tilde{\delta} = \frac{1-\Phi}{1+r}$
   - PE: Liquidity premium $\Phi \rightarrow$ more discounting, less capital than in Aiyagari 94

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4. **Transactions costs, without commitment**
   - firms discount at rate $\tilde{\beta}\tilde{\delta}$, present bias $\tilde{\beta} < 1$
   - PE: less capital than with commitment: $k^n < k^c$

Caveat: for 3. and 4., in GE, r and Φ also change $\rightarrow$ quantitative evaluation

Quantitative evaluation

Calibration

Three sets of parameters:

1. standard or from the literature
2. income process: assume conservative values, do robustness exercises
3. transaction costs: look at the data, consider different values of ϕ

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Parameter	Value	Source
Discount factor β	0.95	Standard
Risk aversion σ	2.00	Standard
Depreciation δ	0.05	Standard
Production weight on labor γ	0.80	Gavazza et al. (2018)
Returns to scale ψ	0.95	Gavazza et al. (2018)
Borrowing limit $\underline{b}$	1.00	Kaplan et al. (2018)
Labor persistence ρ_h	0.50	Conservative, robustness exercises
Labor st dev σ_h	0.03	Conservative, robustness exercises
Transaction cost ϕ	4.00	Data

Data: relative spreads

- Daily data on ordinary shares traded in NYSE (CRSP), relative spreads:

$$RS_{i,t} = \frac{A_{i,t} - B_{i,t}}{0.5(A_{i,t} + B_{i,t})}$$

- 2000Q1 to 2022Q1 (average of daily data), 3k firms, 124k firm-quarter obs

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Relative Spreads, %

	Mean	St. dev.	p10	p50	p90
2000Q1-2022Q1	3.4	2.4	1.5	2.8	5.7

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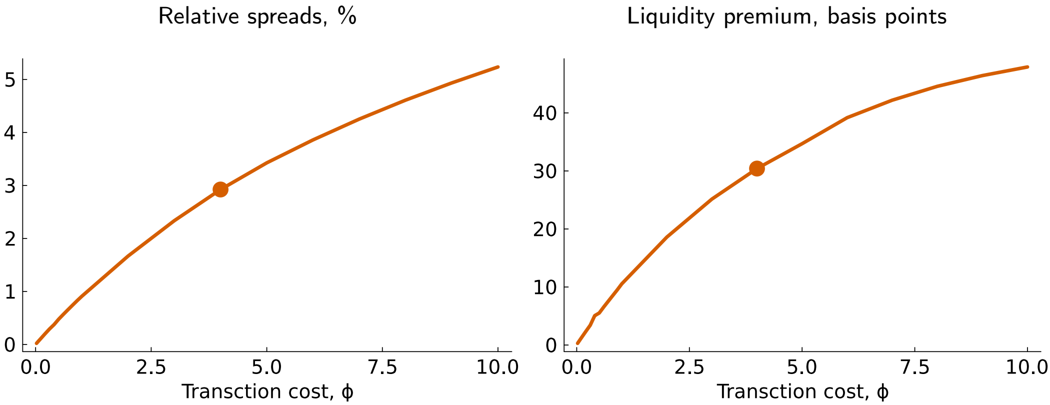
Relative Spreads, %

	Mean	St. dev.	p10	p50	p90
2000Q1-2022Q1	3.4	2.4	1.5	2.8	5.7
2000Q1-2006Q1	3.2	2.3	1.6	2.8	5.2
2010Q1-2019Q4	2.9	1.7	1.5	2.5	4.8

consistent with Næs Skjeltorp Ødegaard (2011) and Corwin Schultz (2012)

▷ histogram ▷ weighted by market cap

Calibration of transaction costs



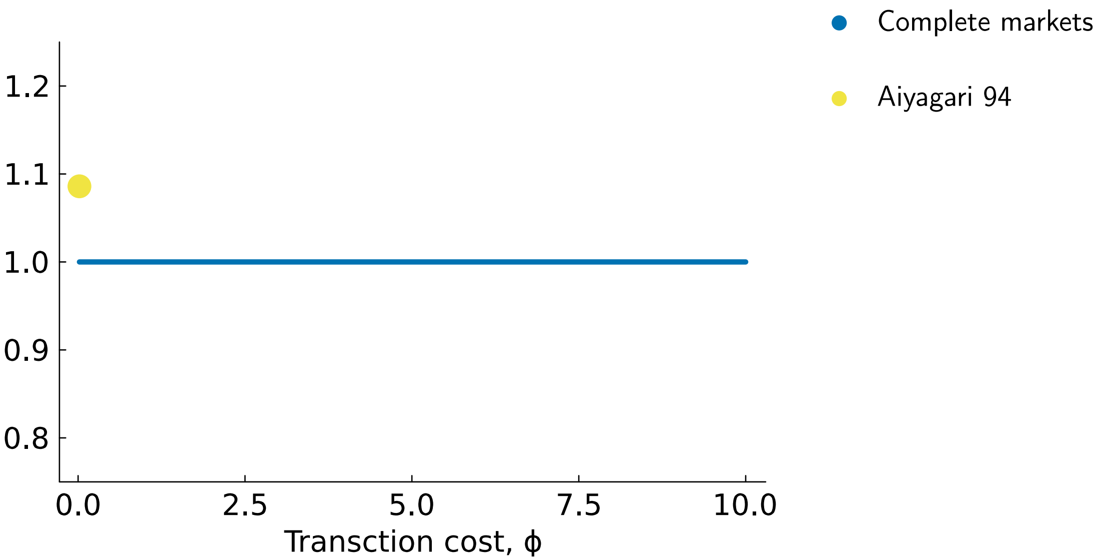
- benchmark calibration: $\phi = 4.0$
- relative spread of 2.9%, consistent with data
- liquidity premium of 30 basis points

Non targeted moments

	Model	Data	Source
Corporate discount rate wedge, percent	1.5	2.1	Gormsen and Huber (2023)
Variance log consumption / variance log income	0.2	0.3	Krueger and Perri (2006)
Illiquid assets to GDP	3.5	2.9	Kaplan et al. (2018)
Liquid assets to GDP	0.5	0.3	Kaplan et al. (2018)
Fraction with $b > 0$	0.5	0.5	Kaplan et al. (2018)
Stock owners at the borrowing constraint, percent	5.4	5.7	SCF 2019

The model is consistent with non-targeted moments despite its stylized nature

Capital, relative to complete markets



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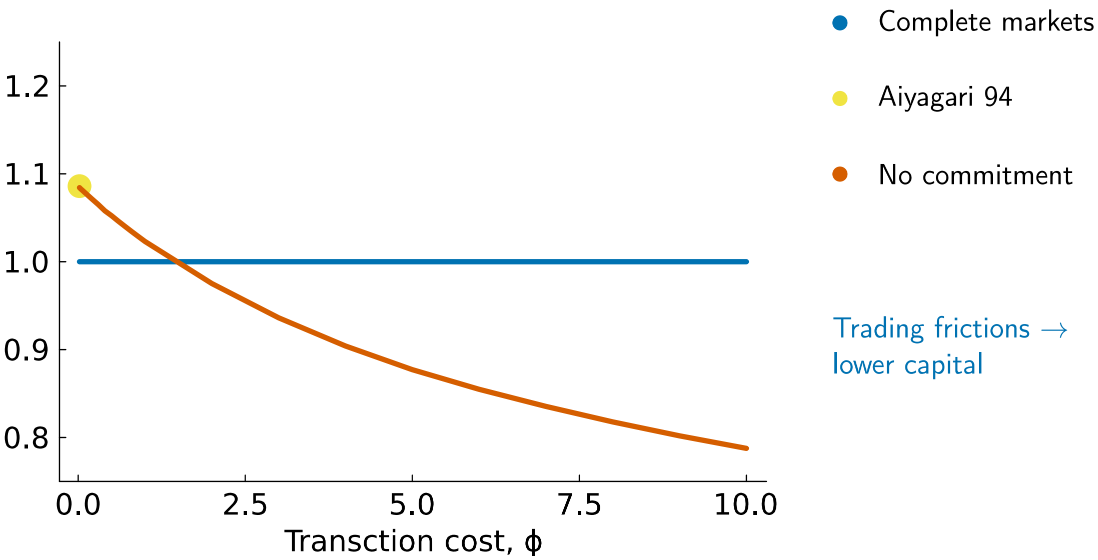
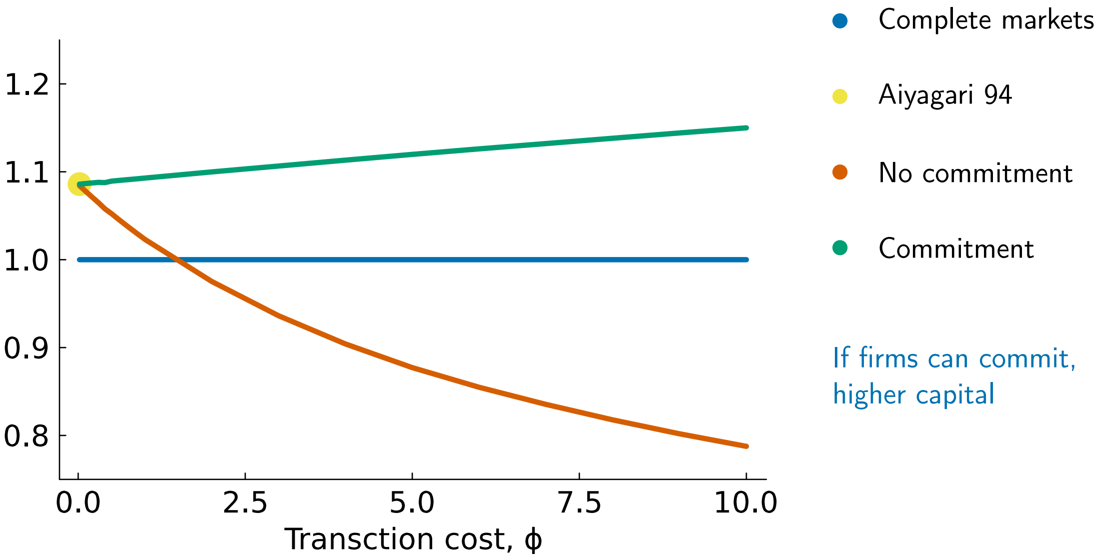


Trading frictions → lower capital

Capital, relative to complete markets



If firms can commit, higher capital

Transmission of trading frictions to investment depends on commitment

With commitment

- SDF: $\tilde{\delta} = \frac{1-\Phi}{1+r}$
- PE: trading frictions depress asset prices ($\uparrow \Phi$) $\rightarrow$ lower level of capital
- GE: higher precautionary savings ($\downarrow r$) $\rightarrow$ larger level of capital
- Quantitatively: moderate increase in capital

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Without commitment

- Present bias: strong force towards more discounting ($\downarrow \tilde{\beta}$) and lower capital

▷ How does the model work?

Extensions & applications

Extensions & applications

1. Empirics: Liquidity & investment in the cross-section
   - Heterogeneous firms → consistent with Amihud Levi (2023), Gormsen Huber (2023)
2. Capital structure: Robust to include corporate bonds
3. Demand of liquidity: Increase in idiosyncratic uncertainty
4. Supply of liquidity: Introduce government bonds
5. Short-termism

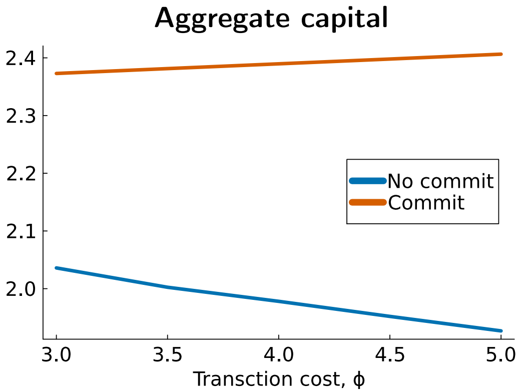
Liquidity & investment in the cross-section

- **Data:** Liquid firms invest more than illiquid ones (Amihud Levi 2023)
- **Model:** extension with two type of firms, liquid and illiquid ones

Liquidity & investment in the cross-section

- **Data:** Liquid firms invest more than illiquid ones (Amihud Levi 2023)
- **Model:** extension with two type of firms, liquid and illiquid ones
- Liquid firm discount rate: $\frac{1}{1+r}$, standard exponential discounting
- Illiquid firm without commitment discount rate: $\frac{1-\bar{\Phi}}{1+r}$
- Liquid firms are more patient $\rightarrow$ have more capital $\rightarrow$ consistent with the data

Liquidity crisis: What happens if ϕ increases?



Effect of liquidity on investment:

With commitment: $\uparrow$ capital (due to increase in precautionary savings, $\downarrow r$)

Without commitment: $\downarrow$ capital (due to illiquid firms)

Cross-sectional evidence is not enough to understand the aggregate effects

Discount rate wedge

- Data (Gormsen Huber 2023):

$$\underbrace{\Lambda}_{\text{Discount rate}} = \underbrace{r^{fin}}_{\text{cost of capital}} + \underbrace{\kappa}_{\text{discount rate wedge}}$$

Discount rate wedge

- Data (Gormsen Huber 2023):

$$\underbrace{\Lambda}_{\text{Discount rate}} = \underbrace{r^{fin}}_{\text{cost of capital}} + \underbrace{\kappa}_{\text{discount rate wedge}}$$

- Model without commitment:

$$\Lambda = -\log \tilde{\beta}\tilde{\delta} = r + \bar{\Phi}$$

- Hence

$$r^{fin} = r + \Phi \qquad \kappa = \bar{\Phi} - \Phi$$

Theory:

1. Model rationalize the discount rate wedge
2. Illiquid firms have higher wedges

Empirics: More illiquid firms have higher discount rate wedges

$$\kappa_{it} = \alpha_t + \delta_i + \beta RS_{i,t} + \gamma X_{i,t} + \varepsilon_{i,t}$$

Relative spread	0.228***	0.184***	0.230***	0.181***
	(0.016)	(0.012)	(0.016)	(0.012)
Observations	27163	27158	27163	27158
R-squared	0.266	0.668	0.266	0.669
FE	Time	Firm, Time	Time	Firm, Time
Controls			Market cap	Market cap

Notes: The dataset is at the firm-quarter level and runs from 2002 to 2021. Standard errors (in parentheses) are clustered by firm. The left-hand side variable is in percent. The regressors are standardized, so that the coefficients estimate the impact of a 1 standard deviation increase. Statistical significance is denoted by *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

- Iliquid firms have higher discount rate wedges
- Model suggests that present bias is a factor behind this empirical finding

▷ Discount rate

Corporate bonds

Firms can borrow at interest rate $1 + r^{cb} = \frac{1+r}{1-\tilde{\phi}}$ up to a limit

- If $\tilde{\phi} < \Phi$ the firm always borrows to the limit independently of its commitment.
- If $\Phi < \tilde{\phi} < \overline{\Phi}$ only the firm without commitment borrows up to the limit.

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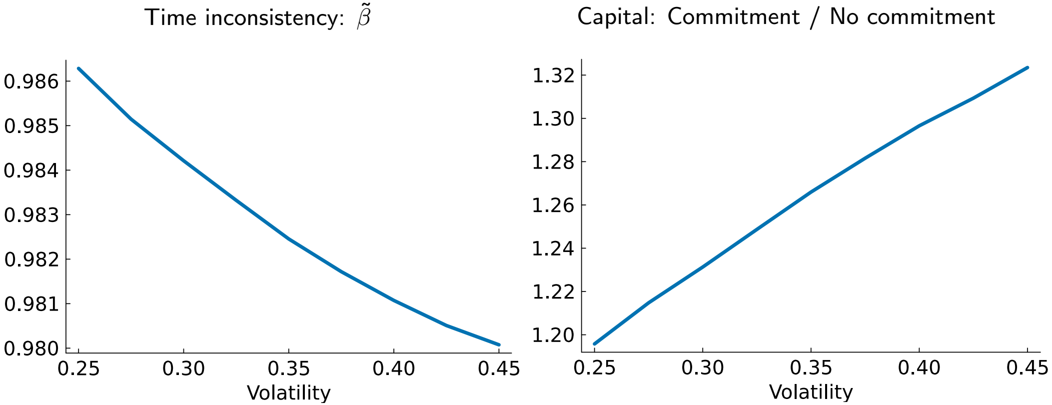
- If $\Phi < \tilde{\phi} < \overline{\Phi}$ only the firm without commitment borrows up to the limit.

Implications:

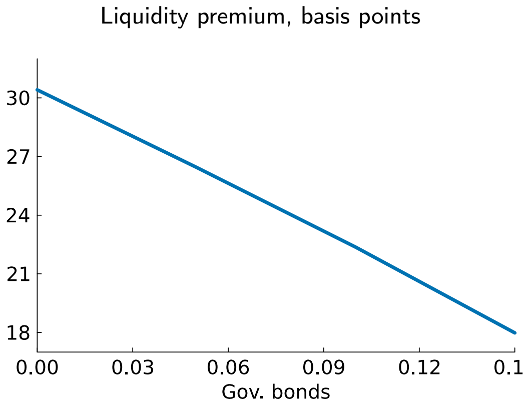
- can alter financing but not investment and the time-inconsistency problem
- firms borrow even if bonds are more illiquid than stocks due to present bias
- rationalize corporate debt that does not rely on the tax advantage of debt

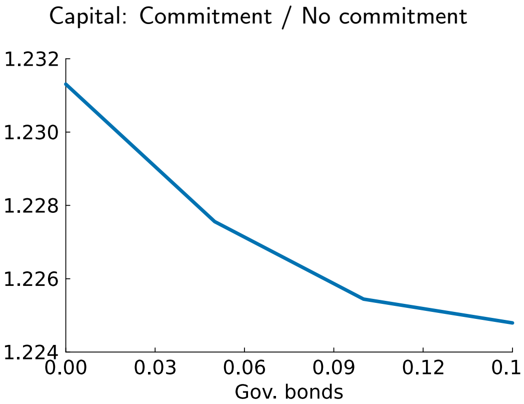
Demand of liquidity: increase idiosyncratic volatility



- Without commitment: more time inconsistency → less capital
- With commitment: more precautionary savings → more capital

Supply of liquidity & government bonds





- Capital closer to complete markets
- Without commitment: less time inconsistency $\rightarrow$ more capital
- With commitment: less precautionary savings $\rightarrow$ less capital

Short-termism

Evidence on short-termism:

- an excessive focus on short-term results at the expense of long-term interests (Graham et al. 05, Terry 23, Fink 15)

- public firms distort their investment to meet short-term targets (Graham et al., 05).

Model: short-termism as a result of (i) trading frictions, and (ii) lack of commitment.

Conclusions

- Aiyagari production economy, with liquid and illiquid assets in general equilibrium
- The problem of the firm is time inconsistent
  - result from frictions in financial markets
  - the discount factor of firms is as if they have quasi-hyperbolic discounting
- Aggregate distortions due to trading frictions depend on commitment
- Rationalize empirical regularities on liquidity and investment

Appendix

Related Literature

- Incomplete markets & firm insurance: Diamond (1967), Dreze (1974), **Grossman Hart (1979)**, Aiyagari Gertler (1991), Heaton Lucas (1996), Magill Quinzii (1996), Espino Kozlowski Sanchez (2018)
  New: Trading frictions and/or GE

- Illiquid assets & macro: Kaplan Violante (2014), Cui Radde (2019), Jeenas Lagos (2020)
  New: Dynamic firm's problem with liquidity frictions

- Hyperbolic discounting: Krusell Smith (2003), Azzimonti (2011), Amador (2012), Cao Werning (2018)
  New: Hyperbolic discounting as a result

- Short-termism: Graham Harvey Rajgopal (2005), Terry (2023)
  New: Don't need additional constraints

Firm: static labor choice

- Static labor choice

$$\max_{l} \left(l^{\gamma} k^{1-\gamma}\right)^{\psi} - wl$$

with labor demand $l = \psi\gamma \frac{y}{w}$

- In equilibrium $w = \psi\gamma k^{(1-\gamma)\psi}$

- Dividends are

$$d_t = F(k_t, k_{t+1}) = zk_t^{\alpha} + (1-\delta)k_t - k_{t+1}$$

where $z = (1-\gamma\psi)\left(\frac{\gamma\psi}{w}\right)^{\frac{\gamma\psi}{1-\gamma\psi}}$ and $\alpha = \frac{(1-\gamma)\psi}{1-\gamma\psi}$

▷ back

Government bonds

- Introduce government bonds
- Lump-sum taxes to pay for the debt services
- Bonds market clearing

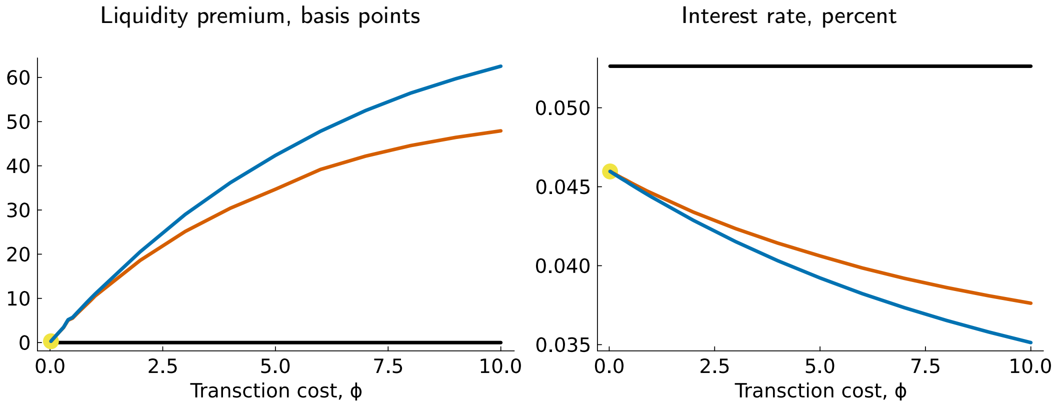
$$\int b'(\theta, b, h) d\Gamma(\theta, b, h) = B^g$$

- As B^g increases: more liquid assets

Public vs private firms

- Asker et al. (2015) finds that public firms invest substantially less than private firms.
- We add private firms to the benchmark equilibrium. Private firms are owned by only one household and are not traded in financial markets.
- The investment decisions of private firms are independent of ϕ, while investment in public firms decreases with the transaction cost.
- For most values of ϕ private firms invest more than public firms, consistent with the empirical evidence.

Commitment: constant discounting

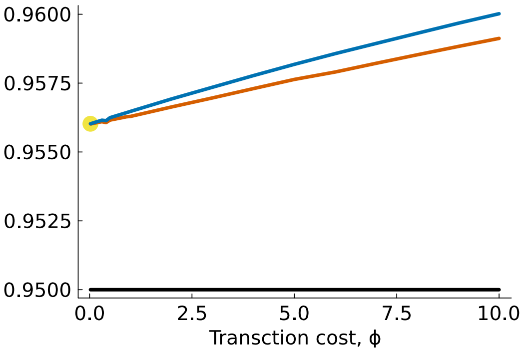


- Higher $\phi \to$ bonds *better* than stocks $\to$ higher liquidity premium & lower r
- Capital with commitment about constant, recall $\tilde{\delta} = \frac{1-\Phi}{1+r}$

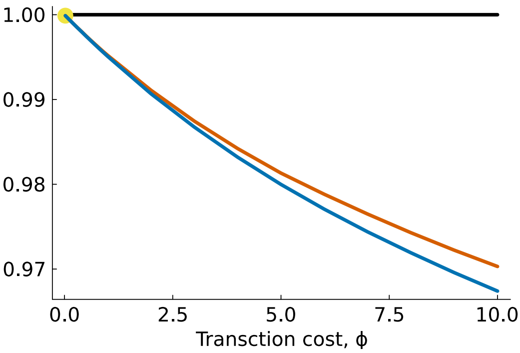
▷ Back

Lack of commitment: quasi-hyperbolic discounting with present bias

$\tilde{\delta} = \frac{1-\Phi}{1+r}$



$\tilde{\beta} = \frac{1-\bar{\Phi}}{1-\Phi}$



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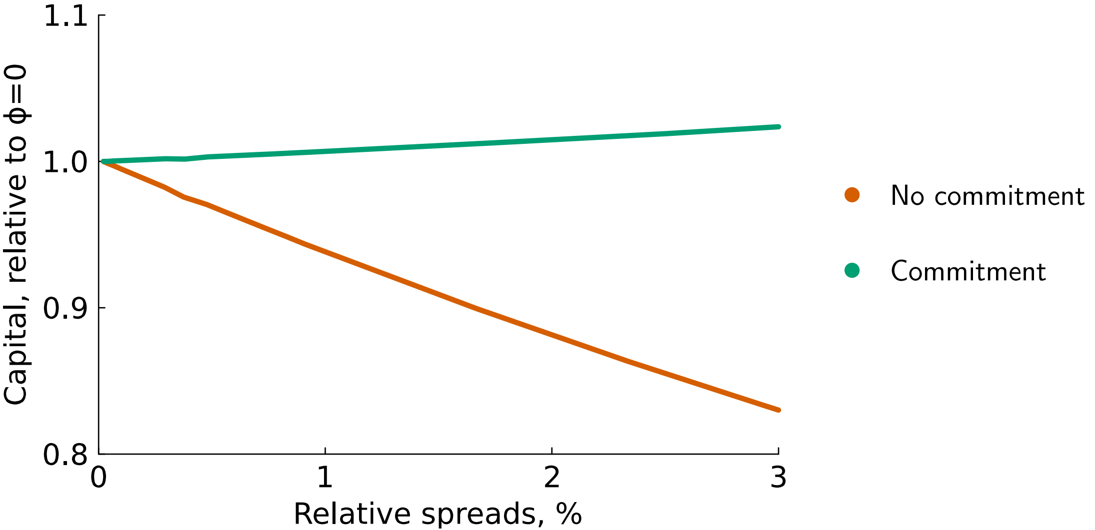
Capital and relative spreads



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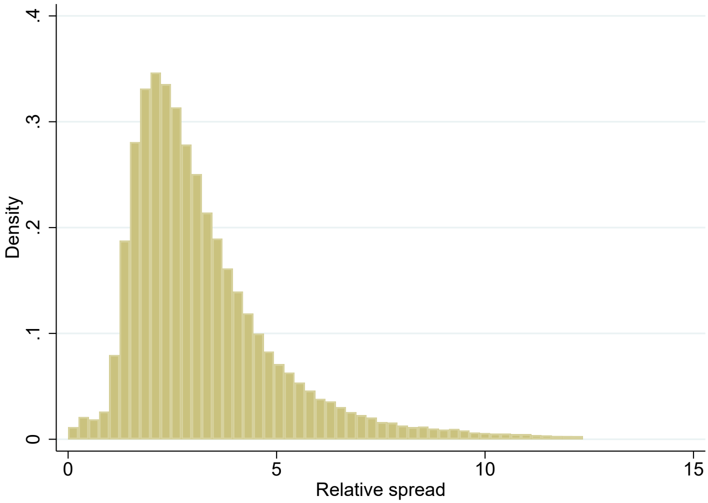
Data: relative spreads, weighted by market capitalization

Relative Spreads, %

	Mean	St. dev.	p10	p50	p90
2000Q1-2022Q1	2.31	1.26	1.24	1.98	3.78
2000Q1-2006Q1	2.64	1.27	1.39	2.35	4.23
2010Q1-2019Q4	1.88	0.8	1.15	1.69	2.84

▷ Back

Relative spreads



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Empirics: More illiquid firms have higher discount rates

Relative spread	0.509***	0.281***	0.497***	0.278***
	(0.026)	(0.016)	(0.027)	(0.016)
Observations	27163	27158	27163	27158
R-squared	0.236	0.805	0.238	0.805
FE	Time	Firm, Time	Time	Firm, Time
Controls			Market cap	Market cap

Notes: The dataset is at the firm-quarter level and runs from 2002 to 2021. Standard errors (in parentheses) are clustered by firm. The left-hand side variable is in percent. The regressors are standardized, so that the coefficients estimate the impact of a 1 standard deviation increase. The specification includes fixed effects for time, or time and firm. Statistical significance is denoted by *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

▷ Back