

Liquidity and Investment in General Equilibrium

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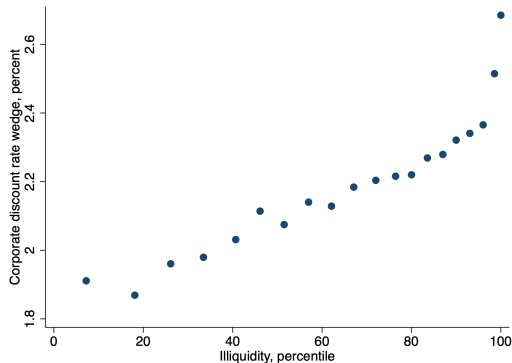
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What is the SDF in an economy with incomplete markets and illiquid assets?

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Corporate Discount Rate Wedge



Fact: Illiquid firms have higher SDF wedges

This paper:

- ▶ Rationalize this fact
- ▶ Implication for investment

Discount rate wedge: Gap between discount rate and cost of capital (Gormsen Huber 2024). Relative spreads from CRSP.

Liquidity and investment in general equilibrium

Model

- ▶ Aiyagari production economy with liquid and illiquid assets in general equilibrium
- ▶ **Firms** take into account that ownership shares trade in **frictional asset markets**

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Results

1. **Theory:** the problem of the firm is **time inconsistent**
 - ▶ firms' SDF as if firms have $\beta - \delta$ **discounting**
 - ▶ result from frictions in financial markets

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Results

1. **Theory:** the problem of the firm is **time inconsistent**
 - ▶ firms' SDF as if firms have $\beta - \delta$ discounting
 - ▶ result from frictions in financial markets
2. **Quantitative:** **trading frictions & aggregate distortions**
 - ▶ Trading frictions have adverse effects on capital **without commitment**
 - ▶ Counterfactual **with commitment**: trading frictions have little effect on capital
3. **Empirics:** rationalize facts on the **cross-section of liquidity, SDF, and investment**

Model

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Aiyagari production economy with liquid and illiquid assets

Households

- ▶ idiosyncratic labor risk h
- ▶ incomplete markets:
 - ▶ liquid bond b , borrowing limit $b \geq \underline{b}$
 - ▶ illiquid stock θ , transaction costs \mathcal{T}


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Firms

- ▶ DRS technology $y = (h^\gamma k^{1-\gamma})^\psi$
- ▶ capital accumulation $k_{t+1} = i_t + (1 - \delta)k_t$  firms solve a **dynamic problem**

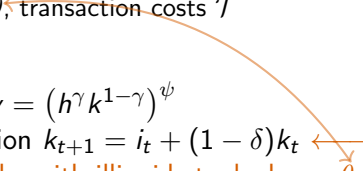
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We study the SDF that firms should use in this economy

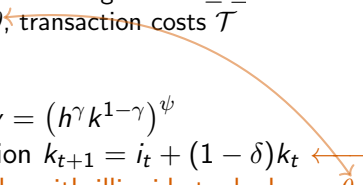
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Stationary equilibrium

- ▶ interest rate r , stock price q , and wage w such that markets clear:

$$\mathbb{E}[b] = 0 \quad \mathbb{E}[\theta] = 1 \quad \mathbb{E}[h] = H$$

Household problem

$$V(\theta, b, h) = \max_{c, b', \Delta^+, \Delta^-} u(c) + \beta \mathbb{E} [V(\theta', b', h')]$$

subject to

$$c + b' + q\Delta^+ \leq wh + b(1+r) + d\theta + q(\Delta^- - \mathcal{T}(\Delta^-))$$

$$\theta' = \theta + \Delta^+ - \Delta^-$$

$$\Delta^- \leq \theta \leftarrow \text{short-selling constraint}$$

$$b' \geq \underline{b} \leftarrow \text{borrowing constraint}$$

$$\mathcal{T}(\Delta^-) = \frac{\phi}{2} (\Delta^-)^2 \leftarrow \text{Transaction costs for sellers (e.g., Heaton Lucas 96)}$$

$$\Delta^+, \Delta^- \geq 0$$

Shareholder's valuation

- ▶ Let $\tilde{q}(\theta, b, h)$ be the shareholder's valuation in units of the consumption good

$$\tilde{q}(\theta, b, h) \equiv \frac{V_{\theta}(\theta, b, h)}{u'(c)}$$

where V_{θ} is the marginal valuation of stocks.

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- ▶ **Lemma:** The shareholder's valuation is

$$\tilde{q}(\theta, b, h) = d + (1 - \phi \Delta^{-}(\theta, b, h)) q$$

- ▶ Buyers, $\Delta^{-} = 0$: agree the value of the firm is $\tilde{q}(\theta, b, h) = d + q$
 - ▶ Sellers: Heterogeneous valuations, depend on marginal transaction cost $\phi \Delta^{-}$
- Disagreement among owners on the valuation of the firm

Firm's problem

Assumption 1: Firm maximizes an ownership-weighted valuation:

$$\int_{\theta, b, h} \theta \underbrace{\left[d + (1 - \phi \Delta^-(\theta, b, h)) q \right]}_{\text{shareholder's valuation}} d\Gamma(\theta, b, h)$$

In spirit of Grossman Hart 1979 (paper also considers Dreze 1974 and DeMarzo 1993).

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Define $\bar{\Phi}$ as the **weighted average marginal transaction cost**

$$\bar{\Phi} \equiv \phi \int_{\theta, b, h} \theta \Delta^-(\theta, b, h) d\Gamma(\theta, b, h)$$

The firm maximizes

$$d + (1 - \bar{\Phi}) q$$

The frictionless case $\phi = 0$

- ▶ The firm's objective is to maximize $d + q$
- ▶ The price is equal to $q = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t d_t$
- ▶ Standard time-consistent problem
- ▶ Maximize the NPV of dividends, discounted at the risk-free rate

→ deviations from *exponential discounting* come from transaction costs, $\phi > 0$

Time Inconsistency in a Three-Period Model

Three-period model

Simplified model to show the **time inconsistency problem**

- ▶ Three periods: $t \in \{0, 1, 2\}$
- ▶ No income risk, two type of households with income $\{H, L, H\}$ and $\{L, H, L\}$
- ▶ No bonds

Three-period model: Euler equations & firm's value

Euler equations:

$$\left(1 - \phi \Delta_0^{j-}\right) q_0 = \beta \frac{u' \left(c_1^j \right)}{u' \left(c_0^j \right)} d_1 + \beta \frac{u' \left(c_1^j \right)}{u' \left(c_0^j \right)} \left(1 - \phi \Delta_1^{j-}\right) q_1$$

$$\left(1 - \phi \Delta_1^{j-}\right) q_1 = \beta \frac{u' \left(c_2^j \right)}{u' \left(c_1^j \right)} d_2$$

Firm's value:

$$\sum_{j \in \{l, h\}} \frac{\theta_0^j}{2} \left[d_0 + (1 - \phi \Delta_0^{j-}) q_0 \right]$$
$$\sum_j \frac{\theta_0^j}{2} \left[d_0 + \beta \frac{u' \left(c_1^j \right)}{u' \left(c_0^j \right)} d_1 + \beta^2 \frac{u' \left(c_2^j \right)}{u' \left(c_0^j \right)} d_2 \right]$$

Time consistency in the three-period model

Problem in period 0

$$\max_{k_1, k_2 \geq 0} \sum_j \frac{\theta_0^j}{2} \left[d_0 + \beta \frac{u'(c_1^j)}{u'(c_0^j)} d_1 + \beta^2 \frac{u'(c_2^j)}{u'(c_0^j)} d_2 \right]$$

Problem in period 1

$$\max_{k_2 \geq 0} \sum_j \frac{\theta_1^j}{2} \left[d_1 + \beta \frac{u'(c_2^j)}{u'(c_1^j)} d_2 \right]$$

The problem is **time consistent** iff the discounting between period 1 and 2 coincides

$$\underbrace{\frac{\sum_j \frac{\theta_0^j}{2} \beta^2 \frac{u'(c_2^j)}{u'(c_0^j)}}{\sum_j \frac{\theta_0^j}{2} \beta \frac{u'(c_1^j)}{u'(c_0^j)}}}_{t=0 \text{ discount between } t=1 \text{ and } t=2} = \underbrace{\sum_j \frac{\theta_1^j}{2} \beta \frac{u'(c_2^j)}{u'(c_1^j)}}_{t=1 \text{ discount between } t=1 \text{ and } t=2}$$

Three-period model, frictionless case $\phi = 0$

The Euler equation implies equalization of marginal rates of substitution across agents:

$$\beta \frac{u'(c_{t+1}^j)}{u'(c_t^j)} = \frac{q_t}{d_{t+1} + q_{t+1}}$$

Hence

$$\underbrace{\frac{\sum_j \frac{\theta_0^j}{2} \beta^2 \frac{u'(c_2^j)}{u'(c_0^j)}}{\sum_j \frac{\theta_0^j}{2} \beta \frac{u'(c_1^j)}{u'(c_0^j)}}}_{t=0 \text{ discount between } t=1 \text{ and } t=2} = \underbrace{\frac{\frac{q_0}{d_1+q_1} \frac{q_1}{d_2+q_2}}{\frac{q_0}{d_1+q_1}}}_{\text{use Euler equation}} = \frac{q_1}{d_2 + q_2} = \underbrace{\sum_j \frac{\theta_1^j}{2} \beta \frac{u'(c_2^j)}{u'(c_1^j)}}_{t=1 \text{ discount between } t=1 \text{ and } t=2}$$

- The problem is time consistent when $\phi = 0$

Three-period model with trading frictions, $\phi > 0$

With transaction costs:

$$\frac{\sum_j \frac{\theta_0^j}{2} \beta^2 \frac{u'(c_2^j)}{u'(c_0^j)}}{\sum_j \frac{\theta_0^j}{2} \beta \frac{u'(c_1^j)}{u'(c_0^j)}} \neq \sum_j \frac{\theta_1^j}{2} \beta \frac{u'(c_2^j)}{u'(c_1^j)}$$

- ▶ The intertemporal marginal rates of substitution are **not** equalized across agents
- ▶ The problem is time inconsistent

Infinite-Horizon Model

Euler equation

$$(1 - \phi \Delta_t^-) q_t = E_t \left[\beta \frac{u'(c_{t+1})}{u'(c_t)} \right] (d_{t+1} + (1 - \Phi_t) q_t) + \eta_t$$

where η_t is the Lagrange multiplier on $\Delta^- \leq \theta$ and

$$\Phi_t \equiv E_t [\phi \Delta_{t+1}^-] + \phi \frac{\text{cov}_t (u'(c_{t+1}), \Delta_{t+1}^-)}{E_t [u'(c_{t+1})]}$$

Φ captures liquidity frictions:

1. Expected marginal transaction costs: $\phi \Delta_{t+1}^- \rightarrow$ lower asset prices
2. Positive covariance if sell in bad times \rightarrow further depress asset prices

The liquidity premium

- ▶ Focus on unconstrained buyers: $\Delta_t^- = 0$, $\Delta_t^+ > 0$, $b_{t+1} > \underline{b}$
- ▶ Asset price:

$$q_t = \frac{d_{t+1} + (1 - \phi^B) q_{t+1}}{1 + r_t}$$

- ▶ The **liquidity premium** is $\phi^B = r^\theta - r$, where r^θ is the yield of the stock

Assumption 2: The firm takes average transaction cost $\bar{\Phi}$ and liquidity premium ϕ^B as given.

Firm's problem

$$V^F(k_t) = \max_{\{k_{t+s}\}_{s \geq 1}} d_t + (1 - \bar{\Phi})q_t$$

subject to

$$q_t = \frac{d_{t+1} + (1 - \Phi)q_{t+1}}{1 + r}$$

where $d_t = F(k_t, k_{t+1}) - zk_t^\alpha - (1 - \delta)k_t = k_{t+1} - k_t$

▷ static labor choice

$\beta - \delta$ discounting and time consistency

$\beta - \delta$ discounting

Proposition: *we can cast the firm's problem as if it has $\beta - \delta$ discounting*

$$V^F(k_t) = \max_{\{k_{t+s}\}_{s \geq 1}} F(k_t, k_{t+1}) + \tilde{\beta} \sum_{s=1}^{\infty} \tilde{\delta}^s F(k_{t+s}, k_{t+s+1})$$

where

- ▶ $\tilde{\delta} = \frac{1-\phi^B}{1+r}$ exponential discounting with liquidity premium
- ▶ $\tilde{\beta} = \frac{1-\bar{\Phi}}{1-\phi^B}$ time-inconsistency
- ▶ $\beta - \delta$ discounting iff $\phi^B \neq \bar{\Phi}$
- ▶ present bias (i.e., $\tilde{\beta} < 1$) iff $\bar{\Phi} > \phi^B$

Time inconsistency & present bias

Proposition: *the difference $\Phi^B - \bar{\Phi}$ is equal to **persistence** and **risk premium** effects:*

$$\Phi^B - \bar{\Phi} = \underbrace{\phi \left(\tilde{\mathbb{E}} \left[\mathbb{E}_t \left[\Delta_{t+1}^- \right] \middle| \text{buyer} \right] - \tilde{\mathbb{E}} \left[\mathbb{E}_t \left[\Delta_{t+1}^- \right] \right] \right)}_{\text{persistence effect}} + \underbrace{\phi \tilde{\mathbb{E}} \left[\frac{\text{cov}_t \left(u' \left(c_{t+1} \right), \Delta_{t+1}^- \right)}{\mathbb{E}_t \left[u' \left(c_{t+1} \right) \right]} \middle| \text{buyer} \right]}_{\text{risk premium}}$$

$\tilde{\mathbb{E}}$ is the cross-sectional expectation, weighted by stock shares θ'

No transaction costs: If $\phi = 0$ then $\Phi^B = \bar{\Phi} = 0$, so $\tilde{\beta} = 1$, time consistent problem.

Intuition: persistence and risk premium

Persistence effect: $\phi \left(\tilde{\mathbb{E}} \left[\mathbb{E}_t \left[\Delta_{t+1}^- \right] \mid \text{buyer} \right] - \tilde{\mathbb{E}} \left[\mathbb{E}_t \left[\Delta_{t+1}^- \right] \right] \right)$

- ▶ difference on average transaction costs for buyers and owners
- ▶ smaller for buyers than owners \rightarrow negative term

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Risk premium: $\phi \tilde{\mathbb{E}} \left[\frac{\text{cov}_t \left(u' \left(c_{t+1} \right), \Delta_{t+1}^- \right)}{\mathbb{E}_t \left[u' \left(c_{t+1} \right) \right]} \middle| \text{buyer} \right]$

- ▶ if sell in bad times \rightarrow positive covariance
- ▶ quantitatively the persistence effect dominates, so $\tilde{\beta} < 1$
- ▶ the problem is **time inconsistent** and the firm has **present bias**

Solution with and without commitment

Solution with and without commitment

With commitment

$$\max_{\{k_{t+s}\}_{s \geq 1}} F(k_t, k_{t+1}) + \tilde{\beta} \sum_{s=1}^{\infty} \tilde{\delta}^s F(k_{t+s}, k_{t+s+1})$$

Steady state

- ▶ SDF: $\tilde{\delta}$
- ▶ Capital

$$k^C = \left(\frac{(1-\gamma)\psi\tilde{\delta}}{1-\tilde{\delta}(1-\delta)} H^{\gamma\psi} \right)^{\frac{1}{1-(1-\gamma)\psi}}$$

Solution with and without commitment

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Without commitment

- ▶ Markov perfect equilibrium

$$\max_{k'} F(k, k') + \tilde{\beta} \tilde{\delta} W(k')$$

$$W(k') = F(k', g(k')) + \tilde{\delta} W(g(k'))$$

Steady state

- ▶ SDF: $\tilde{\beta} \tilde{\delta}$
- ▶ Capital

$$k^N = \left(\frac{(1-\gamma)\psi \tilde{\beta} \tilde{\delta}}{1 - \tilde{\beta} \tilde{\delta}(1-\delta)} H^{\gamma\psi} \right)^{\frac{1}{1-(1-\gamma)\psi}}$$

Incomplete markets, transaction costs, and commitment

Classic results

1. Complete markets

- ▶ $\beta(1+r) = 1$, firms discount at rate $\frac{1}{1+r} = \beta$

2. Aiyagari 94: incomplete markets without transactions costs

- ▶ $\tilde{\beta} = 1$, no problems of commitment
- ▶ firms discount at rate $\frac{1}{1+r}$
- ▶ GE: precautionary savings, $\beta(1+r) < 1$, *more capital* than in complete markets

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New

1. Transactions costs, with commitment

- ▶ firms discount at rate $\tilde{\delta} = \frac{1-\phi^B}{1+r}$
- ▶ PE: Liquidity premium $\phi^B \rightarrow$ more discounting, *less capital* than in Aiyagari 94

Incomplete markets, transaction costs, and commitment

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2. Transactions costs, without commitment

- ▶ firms discount at rate $\tilde{\beta}\tilde{\delta}$, *present bias* $\tilde{\beta} < 1$
- ▶ PE: *less capital than with commitment*: $k^n < k^c$

Caveat: for 3. and 4., in GE, r and ϕ^B also change \rightarrow quantitative evaluation

Quantitative evaluation

Calibration: Transaction costs & Liquidity Premium

Most of the parameters follow a standard calibration

Transaction costs:

Target a **liquidity premium of 35-37 bps**

(van Binsbergen Diamond Grotteria 2022)

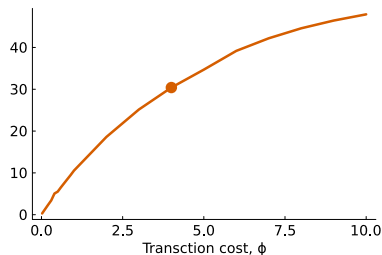
Inferred from call-put parity on S&P 500 options.

Consider $\phi \in [0, 10]$

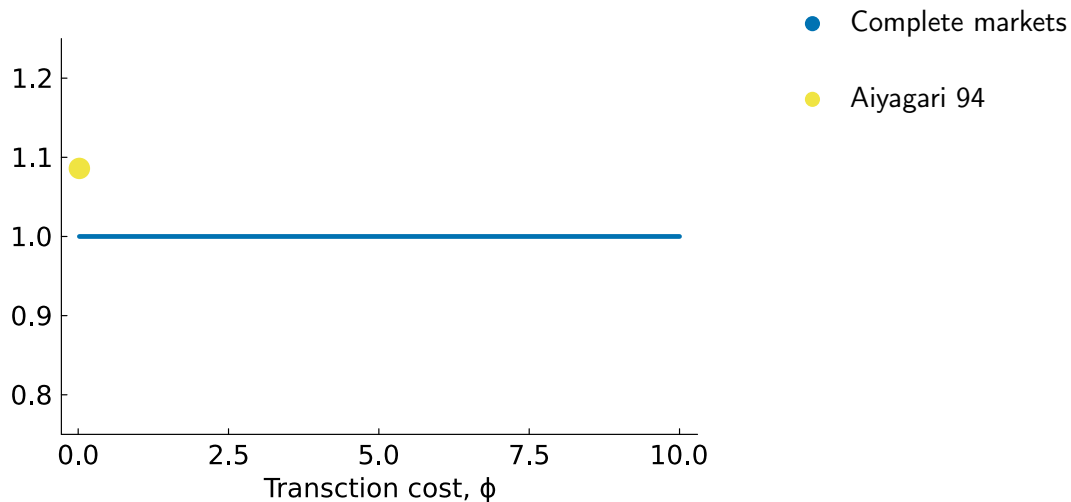
Liquidity premium between 0 and 50 bps

- Calibration
- Non-targeted moments

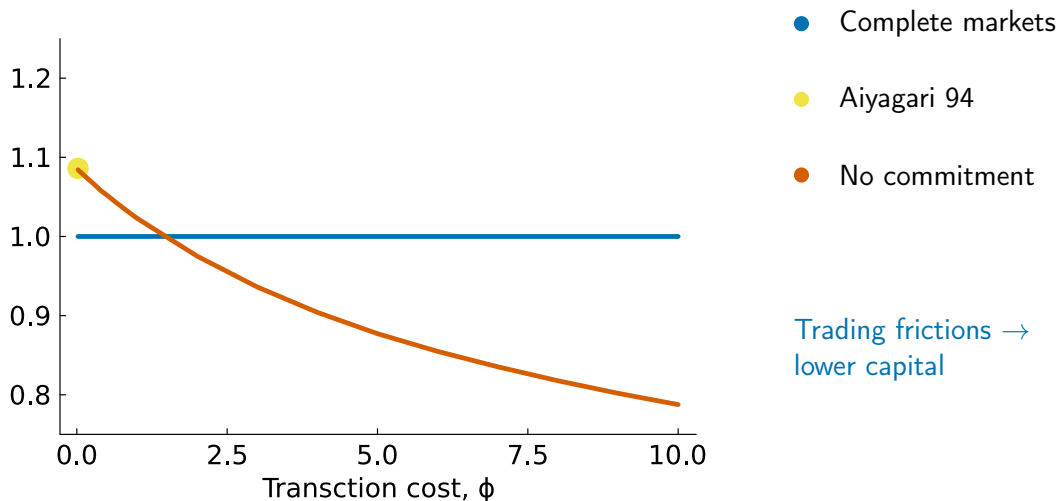
Model: Liquidity premium



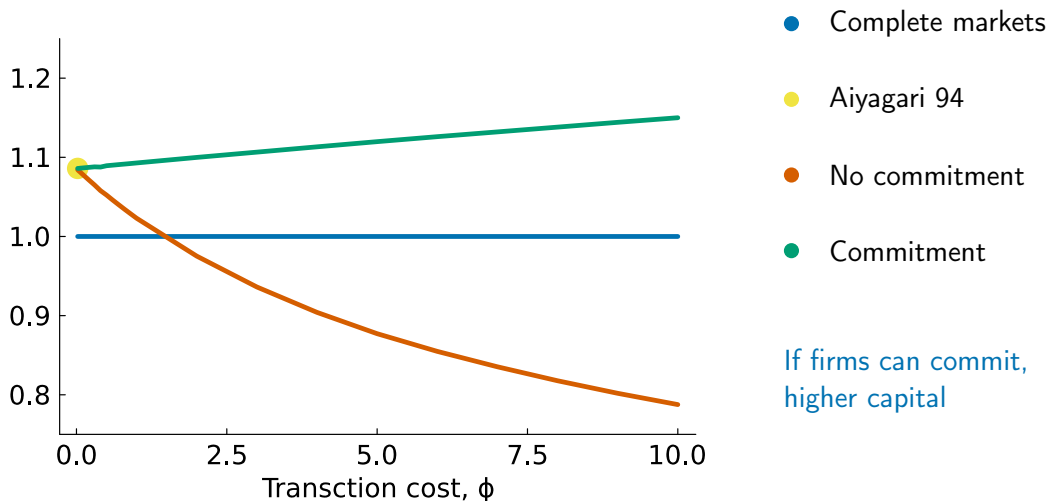
Capital, relative to complete markets



Capital, relative to complete markets



Capital, relative to complete markets



Transmission of trading frictions to investment depends on commitment

With commitment

- ▶ SDF: $\tilde{\delta} = \frac{1-\phi^B}{1+r}$
- ▶ PE: trading frictions depress asset prices ($\uparrow \phi^B$) \rightarrow lower level of capital
- ▶ GE: higher precautionary savings ($\downarrow r$) \rightarrow larger level of capital
- ▶ Quantitatively: moderate increase in capital

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Without commitment

- ▶ Present bias: strong force towards more discounting ($\downarrow \tilde{\beta}$) and lower capital

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Elasticity of capital to the liquidity: An increase of 10 bps in the liquidity premium

- ▶ reduces capital by about 7% **without commitment**
- ▶ increases capital by less than 1% **with commitment**

▷ How does the model work?

Corporate discount rate wedge

- ▶ Gormsen Huber (2024) decompose the firm's discount factor Λ

$$\Lambda = \underbrace{r^{fin}}_{\text{financial cost}} + \underbrace{\kappa}_{\text{discount rate wedge}}$$

- ▶ Model without commitment:

$$r^{fin} \equiv \log \left(\frac{1}{\tilde{\delta}} \right) \approx r + \phi^B, \quad \text{and} \quad \kappa \equiv \log \left(\frac{1}{\tilde{\beta}} \right) \approx \bar{\phi} - \phi^B.$$

- ▶ Present bias generates the discount rate wedge

	Model	Data
Corporate discount rate wedge, percent	1.5	2.1

The model explains about 70% of the wedge

Liquidity and the corporate discount rate wedge

More illiquid firms have higher wedges

$$\kappa_{it} = \alpha_t + \delta_i + \beta \text{liquidity}_{i,t} + \gamma X_{i,t} + \varepsilon_{i,t}$$

Liquidity	0.228*** (0.016)	0.184*** (0.012)	0.230*** (0.016)	0.181*** (0.012)
Observations	27163	27158	27163	27158
R-squared	0.266	0.668	0.266	0.669
FE	Time	Firm, Time	Time	Firm, Time
Controls			Market cap	Market cap

Notes: Firm-quarter data, 2002Q1 to 2021Q4. Standard errors (in parentheses) are clustered by firm. The left-hand side variable is in percent. Liquidity is measured with relative spreads from CRSP. The regressors are standardized, so that the coefficients estimate the impact of a 1 standard deviation increase.

- ▶ Illiquid firms have higher discount rate wedges
- ▶ Model suggests that present bias is a factor behind this empirical finding

▷ Discount rate

Extensions & applications

Extensions & applications

1. Capital structure: Robust to include corporate bonds ▷ Corporate bonds
2. Demand of liquidity: Increase in idiosyncratic uncertainty ▷ Demand
3. Supply of liquidity: Introduce government bonds ▷ Supply
4. Short-termism ▷ Short-termism
5. Heterogeneous firms: Public vs Private ▷ Heterogeneous firms

Conclusions

- ▶ Aiyagari production economy, with liquid and illiquid assets in general equilibrium
- ▶ The problem of the firm is **time inconsistent**
 - ▶ result from frictions in financial markets
 - ▶ the discount factor of firms is as if they have $\beta - \delta$ **discounting**
- ▶ Aggregate distortions due to trading frictions depend on commitment
- ▶ Rationalize **empirical regularities** on liquidity and investment

Appendix

Related Literature

- ▶ **Incomplete markets & firm insurance:** Diamond (1967), Dreze (1974), **Grossman Hart (1979)**, Aiyagari Gertler (1991), Heaton Lucas (1996), Magill Quinzii (1996), Espino Kozlowski Sanchez (2018)
New: Trading frictions and/or GE
- ▶ **Illiquid assets & macro:** Kaplan Violante (2014), Cui Radde (2019), Jeenas Lagos (2020)
New: Dynamic firm's problem with liquidity frictions
- ▶ **$\beta - \delta$ discounting:** Krusell Smith (2003), Azzimonti (2011), Amador (2012), Cao Werning (2018)
New: $\beta - \delta$ discounting as a result
- ▶ **Short-termism:** Graham Harvey Rajgopal (2005), Terry (2023)
New: Don't need additional constraints

Firm: static labor choice

- ▶ Static labor choice

$$\max_l (l^\gamma k^{1-\gamma})^\psi - wl$$

with labor demand $l = \psi\gamma \frac{y}{w}$

- ▶ In equilibrium $w = \psi\gamma k^{(1-\gamma)\psi}$
- ▶ Dividends are

$$d_t = F(k_t, k_{t+1}) = zk_t^\alpha + (1 - \delta)k_t - k_{t+1}$$

where $z = (1 - \gamma\psi) \left(\frac{\gamma\psi}{w}\right)^{\frac{\gamma\psi}{1-\gamma\psi}}$ and $\alpha = \frac{(1-\gamma)\psi}{1-\gamma\psi}$

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Calibration

Parameter	Value
Discount factor β	0.95
Risk aversion σ	2.00
Depreciation δ	0.05
Production weight on labor γ	0.80
Returns to scale ψ	0.95
Borrowing limit \underline{b}	-1.00
Labor persistence ρ_h	0.50
Labor st dev σ_h	0.03
Transaction cost ϕ	4.00

Most of the parameters are standard

Transaction cost: liquidity premium of 40 bps (van Binsbergen Diamond Grotteria 2022)

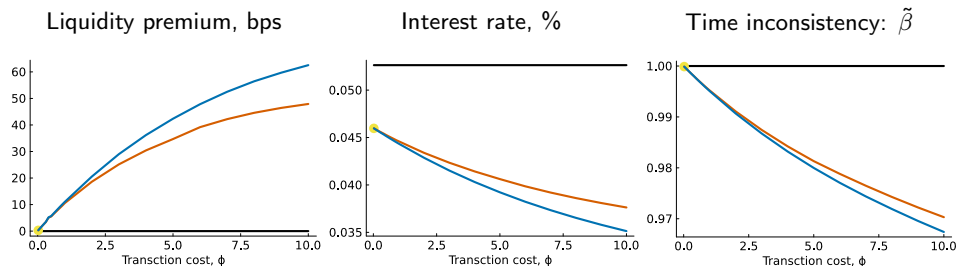
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Non-Targeted Moments

	Model	Data
Corporate discount rate wedge, percent	1.5	2.1
Variance log consumption / variance log income	0.2	0.3
Mean illiquid assets	3.5	2.9
Mean liquid assets	0.5	0.3
Frac. with $b > 0$	0.5	0.5
Stock owners at the borrowing constraint, percent	5.4	5.7

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Commitment: constant discounting



- Higher $\phi \rightarrow$ bonds *better* than stocks \rightarrow higher liquidity premium & lower r
- Capital with commitment about constant, recall $\tilde{\delta} = \frac{1-\phi^B}{1+r}$

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Empirics: More illiquid firms have higher discount rates

Relative spread	0.509*** (0.026)	0.281*** (0.016)	0.497*** (0.027)	0.278*** (0.016)
Observations	27163	27158	27163	27158
R-squared	0.236	0.805	0.238	0.805
FE	Time	Firm, Time	Time	Firm, Time
Controls			Market cap	Market cap

Notes: The dataset is at the firm-quarter level and runs from 2002 to 2021. Standard errors (in parentheses) are clustered by firm. The left-hand side variable is in percent. The regressors are standardized, so that the coefficients estimate the impact of a 1 standard deviation increase. The specification includes fixed effects for time, or time and firm. Statistical significance is denoted by *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

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Corporate bonds

Firms can borrow at interest rate $1 + r^{cb} = \frac{1+r}{1-\tilde{\phi}}$ up to a limit

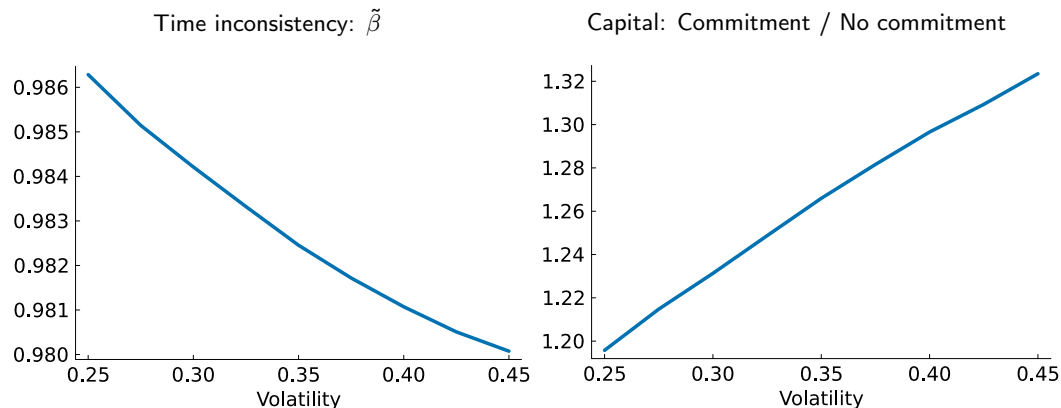
- ▶ If $\tilde{\phi} < \Phi^B$ the firm always borrows to the limit independently of its commitment.
- ▶ If $\Phi^B < \tilde{\phi} < \bar{\Phi}$ only the firm **without commitment** borrows up to the limit.

Implications:

- ▶ can alter financing but not investment and the time-inconsistency problem
- ▶ firms borrow even if bonds are more illiquid than stocks due to present bias
- ▶ rationalize corporate debt that does not rely on the tax advantage of debt

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Demand of liquidity: increase idiosyncratic volatility



- ▶ **Without commitment:** more time inconsistency \rightarrow less capital
- ▶ **With commitment:** more precautionary savings \rightarrow more capital

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Government bonds

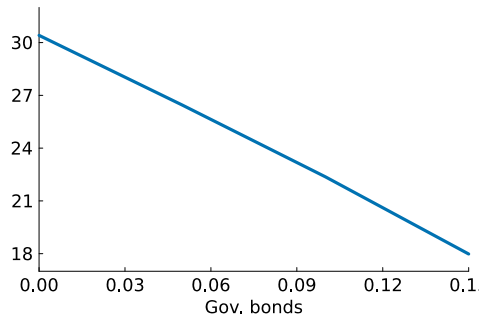
- ▶ Introduce government bonds
- ▶ Lump-sum taxes to pay for the debt services
- ▶ Bonds market clearing

$$\int b'(\theta, b, h) d\Gamma(\theta, b, h) = B^g$$

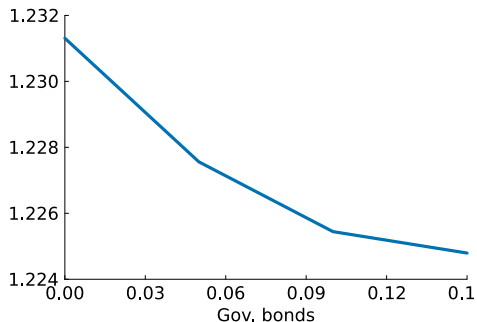
- ▶ As B^g increases: more liquid assets

Supply of liquidity & government bonds

Liquidity premium, basis points



Capital: Commitment / No commitment



- ▶ Capital closer to complete markets
- ▶ Without commitment: less time inconsistency → more capital
- ▶ With commitment: less precautionary savings → less capital

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Short-termism

Evidence on short-termism:

- ▶ an excessive focus on short-term results at the expense of long-term interests (Graham et al. 05, Terry 23, Fink 15)
- ▶ public firms distort their investment to meet short-term targets (Graham et al., 05).

Model: short-termism as a result of (i) trading frictions, and (ii) lack of commitment.

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Heterogeneous Firms: Public vs private firms

- ▶ Asker et al. (2015) finds that public firms invest substantially less than private firms.
- ▶ We add private firms to the benchmark equilibrium. Private firms are owned by only one household and are not traded in financial markets.
- ▶ The investment decisions of private firms are independent of ϕ , while investment in public firms decreases with the transaction cost.
- ▶ For most values of ϕ private firms invest more than public firms, consistent with the empirical evidence.