# Liquidity and Investment in General Equilibrium

Nicolas Caramp UC Davis Julian Kozlowski St. Louis Fed Keisuke Teeple U Waterloo

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# Investment and liquidity

- A central question in macroeconomics concerns the determinants of investment.
- Compare marginal value of firms' capital with replacement cost (Tobin, 1969).
- Result when owners agree that firm should maximize cum-dividend value.
  - ▶ E.g. neoclassical model with complete markets or representative agent.
- But what happens if owners disagree on the firm's optimal strategy?

# Investment and liquidity

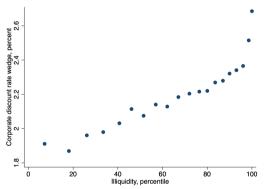
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# This paper: Liquidity as a source of disagreement

- ► Empirically relevant channel Amihud Mendelson Pedersen (2005)
- ► Central feature of new wave of macro models Kaplan Violante 2014, HANK

# Discount rates and liquidity

# Corporate Discount Rate Wedge



#### Fact

Illiquid firms have higher wedges

# This paper

A theory that rationalizes this fact. Study the implication for investment.

Discount rate wedge: Gap between discount rate and cost of capital

(Gormsen and Huber, 2025). Relative bid-ask spreads from CRSP.

# Liquidity and investment in general equilibrium

#### Model

- Aiyagari production economy with liquid and illiquid assets in general equilibrium
- Firms take into account that ownership shares trade in frictional asset markets

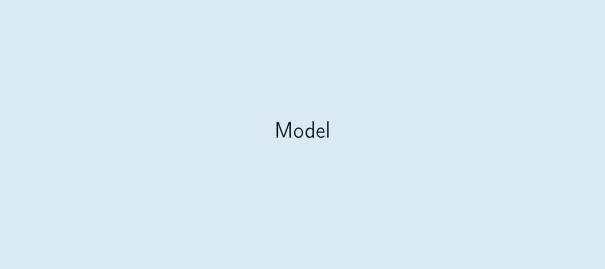
# Liquidity and investment in general equilibrium

#### Model

- Aiyagari production economy with liquid and illiquid assets in general equilibrium
- Firms take into account that ownership shares trade in frictional asset markets

#### Results

- 1. Theory: the problem of the firm is time inconsistent
  - firms' SDF as if firms have  $\beta \delta$  discounting
  - This result from frictions in financial markets
- 2. Quantitative: trading frictions & aggregate distortions
  - ► Trading frictions have adverse effects on capital without commitment
  - ► Counterfactual with commitment: trading frictions have little effect on capital
- 3. Empirics: rationalize facts on the cross-section of liquidity, SDF, and investment



# Model: Aiyagari production economy with liquid and illiquid assets

#### Households

Idiosyncratic labor risk *h*. incomplete markets:

- liquid bond b, borrowing limit  $b' \ge \underline{b}$
- ightharpoonup illiquid stock  $\theta$ , transaction costs  $\mathcal{T}$

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#### **Firms**

Technology  $y_t = h_t^{\gamma} k_t^{1-\gamma}$ Capital accumulation  $k_{t+1} = i_t + (1-\delta)k_t$ Ownership through illiquid stock shares  $\theta$ 

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**Stationary equilibrium**: interest rate r, stock price q, and wage w such that markets clear:

$$\mathbb{E}[b] = 0$$
  $\mathbb{E}[\theta] = 1$   $\mathbb{E}[h] = H$ 

We analyze the SDF that firms use in this setting

# Household problem

$$V(\theta, b, h) = \max_{c, b', \Delta^{+}, \Delta^{-}} u(c) + \beta \mathbb{E} \left[ V \left( \theta', b', h' \right) \right]$$

subject to

$$c + b' + q\Delta^{+} \leq wh + b(1+r) + d\theta + q\left(\Delta^{-} - \mathcal{T}\left(\Delta^{-}\right)\right)$$

$$\theta' = \theta + \Delta^{+} - \Delta^{-}$$

$$\Delta^{-} \leq \theta \leftarrow \text{short-selling constraint}$$

$$b' \geq \underline{b} \leftarrow \text{borrowing constraint}$$

$$\mathcal{T}\left(\Delta^{-}\right) = \frac{\phi}{2}\left(\Delta^{-}\right)^{2} \leftarrow \text{Transaction costs for sellers (e.g., Heaton Lucas 96)}$$

$$\Delta^{+}, \Delta^{-} > 0$$

#### Shareholder's valuation

Let  $\tilde{q}(\theta, b, h)$  be the shareholder's valuation in units of the consumption good

$$\tilde{q}\left(\theta,b,h\right)\equiv\frac{V_{\theta}\left(\theta,b,h\right)}{u'\left(c\right)}$$

where  $V_{\theta}$  is the marginal valuation of stocks.

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#### Lemma

The shareholder's valuation is

$$\tilde{q}\left( heta,b,h
ight)=d+\left(1-\phi\Delta^{-}\left( heta,b,h
ight)
ight)q$$

- Buyers,  $\Delta^- = 0$ : agree the value of the firm is  $\tilde{q}(\theta, b, h) = d + q$
- **Sellers:** Heterogeneous valuations, depend on marginal transaction cost  $\phi \Delta^-$
- ightarrow Disagreement among owners on the valuation of the firm

# Firm's problem

#### Assumption 1

Firm maximizes an ownership-weighted valuation:

$$\int_{\theta,b,h} \theta \underbrace{\left[d + (1 - \phi \Delta^{-}(\theta,b,h))q\right]}_{\text{shareholder's valuation}} d\Gamma(\theta,b,h)$$

In spirit of Grossman and Hart (1979) (paper also considers DeMarzo, 1993; Dreze, 1974).

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Define  $\bar{\Phi}$  as the weighted average marginal transaction cost

$$ar{\Phi} \equiv \phi \int_{ heta,b,h} heta \Delta^-( heta,b,h) d\Gamma( heta,b,h)$$

The firm maximizes  $d + (1 - \bar{\Phi}) q$ 

# The frictionless case $\phi = 0$

- ▶ The firm's objective is to maximize d + q
- ▶ The price is equal to  $q = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t d_t$
- Standard time-consistent problem
- Maximize the NPV of dividends, discounted at the risk-free rate

#### Result

Deviations from exponential discounting come from transaction costs:  $\phi>0$ 

Time Inconsistency in a Three-Period Model

# Three-period model

Simplified model to show the time inconsistency problem

▶ Three periods:  $t \in \{0, 1, 2\}$ 

No income risk, two type of households with income  $\{H, L, H\}$  and  $\{L, H, L\}$ 

No bonds

# Three-period model: Euler equations & firm's value

Euler equations:

$$\left(1-\phi\Delta_0^{j-}
ight)q_0=etarac{u'\left(c_1^j
ight)}{u'\left(c_0^j
ight)}d_1+etarac{u'\left(c_1^j
ight)}{u'\left(c_0^j
ight)}\left(1-\phi\Delta_1^{j-}
ight)q_1 \ \left(1-\phi\Delta_1^{j-}
ight)q_1=etarac{u'\left(c_2^j
ight)}{u'\left(c_1^j
ight)}d_2$$

Firm's value:

$$egin{aligned} \sum_{j \in \left\{I,h
ight\}} rac{ heta_0^j}{2} \left[d_0 + (1-\phi\Delta_0^{j-})q_0
ight] \ \sum_j rac{ heta_0^j}{2} \left[d_0 + eta rac{u'\left(c_1^j
ight)}{u'\left(c_0^j
ight)} d_1 + eta^2 rac{u'\left(c_2^j
ight)}{u'\left(c_0^j
ight)} d_2
ight] \end{aligned}$$

# Time consistency in the three-period model

#### Problem in period 0

$$\max_{k_1,k_2 \geq 0} \sum_j \frac{\theta_0^j}{2} \left[ d_0 + \beta \frac{u'\left(c_1^j\right)}{u'\left(c_0^j\right)} d_1 + \beta^2 \frac{u'\left(c_2^j\right)}{u'\left(c_0^j\right)} d_2 \right]$$

# Problem in period 1

$$\max_{k_2 \geq 0} \sum_j \frac{\theta_1^j}{2} \left[ d_1 + \beta \frac{ \textcolor{red}{u'} \left( \textcolor{blue}{c_2^j} \right)}{\textcolor{blue}{u'} \left( \textcolor{blue}{c_1^j} \right)} d_2 \right]$$

The problem is time consistent iff the discounting between period 1 and 2 coincides

$$\frac{\sum_{j} \frac{\theta_{0}^{j}}{2} \beta^{2} \frac{u'(c_{2}^{j})}{u'(c_{0}^{j})}}{\sum_{j} \frac{\theta_{0}^{j}}{2} \beta \frac{u'(c_{1}^{j})}{u'(c_{0}^{j})}} = \sum_{j} \frac{\theta_{1}^{j}}{2} \beta \frac{u'(c_{2}^{j})}{u'(c_{1}^{j})}$$

$$t = 0 \text{ discount between } t = 1 \text{ and } t = 2$$

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# Three-period model, frictionless case $\phi = 0$

The Euler equation implies equalization of marginal rates of substitution across agents:

$$\beta \frac{u'\left(c_{t+1}^{j}\right)}{u'\left(c_{t}^{j}\right)} = \frac{q_{t}}{d_{t+1} + q_{t+1}}$$

Hence

$$\frac{\sum_{j} \frac{\theta_0^j}{2} \beta^2 \frac{u'(c_2^j)}{u'(c_0^j)}}{\sum_{j} \frac{\theta_0^j}{2} \beta \frac{u'(c_1^j)}{u'(c_0^j)}} = \underbrace{\frac{q_0}{\frac{d_1+q_1}{d_2+q_2}} \frac{q_1}{\frac{q_0}{d_1+q_1}}}_{\text{use Euler equation}} = \frac{q_1}{d_2+q_2} = \underbrace{\sum_{j} \frac{\theta_1^j}{2} \beta \frac{u'(c_2^j)}{u'(c_1^j)}}_{t=1 \text{ discount between } t=1 \text{ and } t=2}$$

▶ The problem is time consistent when  $\phi = 0$ 

# Three-period model with trading frictions, $\phi > 0$

#### With transaction costs:

$$\frac{\sum_{j} \frac{\theta_{0}^{j}}{2} \beta^{2} \frac{u'(c_{2}^{j})}{u'(c_{0}^{j})}}{\sum_{j} \frac{\theta_{0}^{j}}{2} \beta \frac{u'(c_{1}^{j})}{u'(c_{0}^{j})}} \neq \sum_{j} \frac{\theta_{1}^{j}}{2} \beta \frac{u'(c_{2}^{j})}{u'(c_{1}^{j})}$$

▶ The intertemporal marginal rates of substitution are **not** equalized across agents

► The problem is time inconsistent

Infinite-Horizon Model

# Euler equation

### **Euler Equation**

$$(1 - \phi \Delta_t^-)q_t = E_t \left[\beta \frac{u'(c_{t+1})}{u'(c_t)}\right] (d_{t+1} + (1 - \Phi_t) q_{t+1}) + \eta_t$$

where  $\eta_t$  is the Lagrange multiplier on  $\Delta^- \leq \theta$  and  $\Phi$  captures liquidity frictions:

$$\Phi_{t} \equiv \mathbb{E}_{t} \left[ \phi \Delta_{t+1}^{-} \right] + \phi \frac{\mathsf{cov}_{t} \left( u'(c_{t+1}), \Delta_{t+1}^{-} \right)}{\mathbb{E}_{t} \left[ u'(c_{t+1}) \right]}$$

- 1. Expected marginal transaction costs:  $\phi\Delta_{t+1}^- o$  lower asset prices
- 2. Positive covariance if sell in bad times  $\rightarrow$  further depress asset prices

# The liquidity premium

Focus on unconstrained buyers:  $\Delta_t^-=0$ ,  $\Delta_t^+>0$ ,  $b_{t+1}>\underline{b}$ 

#### Asset price

$$q_t = rac{d_{t+1} + \left(1 - rac{\Phi^B}{}
ight)q_{t+1}}{1 + r_t}$$

The liquidity premium is  $\Phi^B = r^{\theta} - r$ , where  $r^{\theta}$  is the yield of the stock

#### Assumption 2

The firm takes average transaction cost  $\bar{\Phi}$  and the liquidity premium  $\Phi^B$  as given.

# Firm's problem

A firm with commitment solves:

$$V^F(k_t) = \max_{\{k_{t+s}\}_{s\geq 1}} d_t + (1-ar{\Phi})q_t$$

subject to

$$q_t = rac{d_{t+1} + (1 - \Phi) \, q_{t+1}}{1 + r}$$

where 
$$d_t = F(k_t, k_{t+1}) = zk_t + (1 - \delta)k_t - k_{t+1}$$

# $eta-\delta$ discounting and time consistency

# $\beta - \delta$ discounting

#### Proposition

We can cast the firm's problem as if it has  $\beta - \delta$  discounting

$$V^{F}(k_{t}) = \max_{\{k_{t+s}\}_{s \geq 1}} F(k_{t}, k_{t+1}) + \frac{\tilde{\beta}}{\tilde{\beta}} \sum_{s=1}^{\infty} \tilde{\delta}^{s} F(k_{t+s}, k_{t+s+1})$$

#### where

- $ilde{\delta} = rac{1 \Phi^B}{1 + r}$  exponential discounting with liquidity premium
- $\tilde{\beta} = \frac{1-\bar{\Phi}}{1-\Phi^B}$  time-inconsistency
- $eta-\delta$  discounting iff  $\Phi^B
  eq ar\Phi$  , and present bias (i.e., ildeeta<1) iff  $ar\Phi>\Phi^B$

# Time inconsistency & present bias

#### Proposition

The difference  $\Phi^B - \bar{\Phi}$  is equal to persistence and risk premium effects:

$$\Phi^{B} - \bar{\Phi} = \phi \left( \tilde{\mathbb{E}} \left[ \mathbb{E}_{t} \left[ \Delta_{t+1}^{-} \right] \middle\| \text{ buyer} \right] - \tilde{\mathbb{E}} \left[ \mathbb{E}_{t} \left[ \Delta_{t+1}^{-} \right] \right] \right)$$
persistence effect
$$+ \phi \tilde{\mathbb{E}} \left[ \left. \frac{\text{cov}_{t} \left( u' \left( c_{t+1} \right), \Delta_{t+1}^{-} \right)}{\mathbb{E}_{t} \left[ u' \left( c_{t+1} \right) \right]} \middle\| \text{ buyer} \right] \right]$$
risk premium

 $\tilde{\mathbb{E}}$  is the cross-sectional expectation, weighted by stock shares  $\theta'$ 

No transaction costs: If  $\phi = 0$  then  $\Phi^B = \bar{\Phi} = 0$ , so  $\tilde{\beta} = 1$ , time consistent problem.

# Intuition: persistence and risk premium

#### Persistence effect

$$\phi\left(\tilde{\mathbb{E}}\left[\left.\mathbb{E}_{t}\left[\Delta_{t+1}^{-}\right]\right\|\operatorname{\mathsf{buyer}}\right]-\tilde{\mathbb{E}}\left[\mathbb{E}_{t}\left[\Delta_{t+1}^{-}\right]\right]\right)$$

Difference on average transaction costs for buyers and owners  $\text{Smaller for buyers than owners} \to \text{negative term}$ 

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Difference on average transaction costs for buyers and owners  $\text{Smaller for buyers than owners} \to \text{negative term}$ 

#### Risk premium

$$\phi \widetilde{\mathbb{E}} \left[ \left. \begin{array}{c} \mathsf{cov}_t \left( u' \left( c_{t+1} \right), \Delta_{t+1}^- \right) \\ \mathbb{E}_t \left[ u' \left( c_{t+1} \right) 
ight] \end{array} \right| \mathsf{buyer} \right]$$

If sell in bad times  $\rightarrow$  positive covariance

- lacktriangle Quantitatively, the persistence effect dominates, so  $ilde{eta} < 1$
- ► The problem is time inconsistent and the firm has present bias

# Solution with and without commitment

# Solution with and without commitment

#### With commitment

$$\max_{\{k_{t+s}\}_{s\geq 1}} F(k_t, k_{t+1}) + \frac{\tilde{\beta}}{\tilde{\beta}} \sum_{s=1}^{\infty} \tilde{\delta}^s F(k_{t+s}, k_{t+s+1})$$

#### Capital with commitment

$$k^{C}=\left(rac{\left(1-\gamma
ight) ilde{\delta}}{1- ilde{\delta}\left(1-\delta
ight)}H^{\gamma}
ight)^{rac{1}{\gamma}}$$

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ight) ilde{\delta}}{1- ilde{\delta}\left(1-\delta
ight)}H^{\gamma}
ight)^{rac{1}{\gamma}}$$

#### Without commitment

Markov perfect equilibrium

$$\max_{k'} F(k, k') + \frac{\tilde{eta}}{\tilde{\delta}} \tilde{\delta} W(k')$$
 $W(k') = F(k', g(k')) + \frac{\tilde{\delta}}{\tilde{\delta}} W(g(k'))$ 

#### Capital without commitment

$$k^{\mathcal{N}} = \left(rac{\left(1-\gamma
ight) ilde{oldsymbol{eta}} ilde{\delta}}{1- ilde{oldsymbol{eta}} ilde{\delta}\left(1-\delta
ight)}H^{\gamma}
ight)^{rac{1}{\gamma}}$$

# Incomplete markets, transaction costs, and commitment

#### Classic results

- **Complete markets**:  $\beta(1+r)=1$ , firms discount at rate  $\frac{1}{1+r}=\beta$
- ▶ Aiyagari 94: incomplete markets without transactions costs
  - $ilde{eta}=1$  , no problems of commitment
  - firms discount at rate  $\frac{1}{1+r}$
  - ▶ GE: precautionary savings,  $\beta(1+r) < 1$ , over accumulation of capital

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#### New results

- ► Transaction costs, with commitment
  - firms discount at rate  $\tilde{\delta} = \frac{1 \Phi^B}{1 + r}$
  - $\triangleright$  PE: Liquidity premium  $\Phi^B \rightarrow$  more discounting, less capital
- Transaction costs, without commitment
  - firms discount at rate  $\tilde{\beta}\tilde{\delta}$ , present bias  $\tilde{\beta} < 1$
  - **PE**: less capital than with commitment:  $k^n < k^c$

Quantitative evaluation

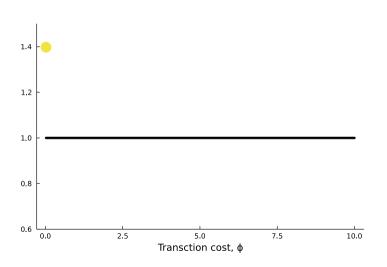
### Calibration: Standard Parameters

Most of the parameters are standard in the literature.

Parameter	Value	Target
Discount factor $\beta$	0.95	
Risk aversion $\sigma$	2.00	
Depreciation $\delta$	0.05	
Labor share $\gamma$	0.66	
Labor autoregressive coefficient $\rho_h$	0.91	Floden Linde (2001)
Labor innovation varianc $\sigma_h^2$	0.04	Floden Linde (2001)
Borrowing limit <u>b</u>	-0.59	Household unsecured credit-to-GDP of 17%
Transaction cost $\phi$	3.38	Liquidity premium of 37 bps (van Binsbergen et al., 2022)

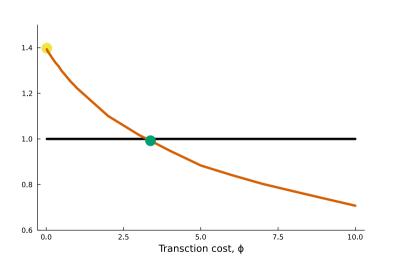
#### ▶ Moments

### Capital, relative to complete markets



- Complete markets
- Aiyagari 94

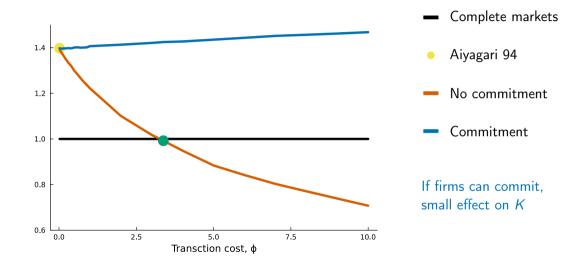
### Capital, relative to complete markets



- Complete markets
- Aiyagari 94
- No commitment

Trading frictions  $\rightarrow$  lower capital

### Capital, relative to complete markets



### Transmission of trading frictions to investment depends on commitment

#### With commitment

- SDF:  $\tilde{\delta} = \frac{1 \Phi^B}{1 + r}$
- ▶ PE: trading frictions depress asset prices  $(\uparrow \Phi^B)$  → lower level of capital
- ▶ GE: higher precautionary savings  $(\downarrow r)$  → larger level of capital
- Quantitatively: moderate increase in capital

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#### With commitment

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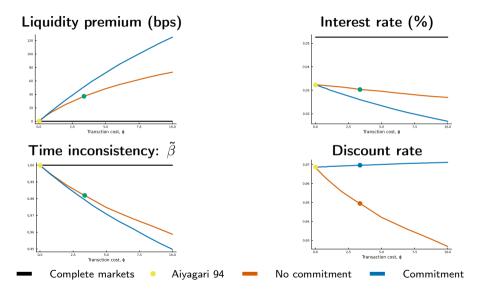
#### Without commitment

Present bias: strong force towards more discounting  $(\downarrow \tilde{\beta})$  and lower capital

#### Elasticity of capital to liquidity: A 10 bps increase of the liquidity premium

- reduces capital by 9.2% without commitment
- ▶ increases capital by 0.4% with commitment

#### Discount factors



Extensions & applications

### Extensions & applications

1. Corporate discount rate wedge ▶ Wedge 2. Capital structure: Robust to include corporate bonds 3. Demand of liquidity: Increase in idiosyncratic uncertainty ▶ Demand 4. Supply of liquidity: Introduce government bonds Supply 5. Disagreement from capital gains tax 6. Short-termism Short-termism
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 7. Heterogeneous firms: Public vs Private ▶ Heterogeneous firms

#### Conclusions

- Aiyagari production economy, with liquid and illiquid assets in general equilibrium
- The problem of the firm is time inconsistent
  - ► This result arises from frictions in financial markets
  - the discount factor of firms is as if they have  $\beta \delta$  discounting

Aggregate distortions due to trading frictions depend on commitment

Rationalize empirical regularities on liquidity and investment

# Appendix

#### Related Literature

- ▶ Incomplete markets & firm insurance: Diamond (1967), Dreze (1974), Grossman Hart (1979), Aiyagari Gertler (1991), Heaton Lucas (1996), Magill Quinzii (1996), Espino Kozlowski Sanchez (2018)
  New: Trading frictions and/or GE
- Illiquid assets & macro: Kaplan Violante (2014), Cui Radde (2019), Jeenas Lagos (2020)
   New: Dynamic firm's problem with liquidity frictions
- $eta \delta$  discounting: Krusell Smith (2003), Azzimonti (2011), Amador (2012), Cao Werning (2018) New:  $eta - \delta$  discounting as a result
- ► Short-termism: Graham Harvey Rajgopal (2005), Terry (2023) New: Don't need additional constraints

### Firm: static labor choice

Static labor choice

$$\max_{l} I^{\gamma} k^{1-\gamma} - wI$$

with labor demand  $\mathit{I} = \gamma \frac{\mathit{y}}{\mathit{w}}$ 

- In equilibrium  $w = \gamma k^{1-\gamma}$
- Dividends are

$$d_t = F(k_t, k_{t+1}) = zk_t + (1 - \delta)k_t - k_{t+1}$$

where 
$$z = (1 - \gamma) \left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1 - \gamma}}$$

▷ back

#### Model and data moments

	Model	Data
Target		
Liquidity premium, bps	37	37
Credit to GDP, percent	17	17
Non-target		
Corporate discount rate wedge, percent	1.8	2.1
Capital to GDP	3.3	3.0

Note: Liquidity premium from van Binsbergen et al. (2022), credit to GDP from Flow of Funds tables, corporate discount rates from Gormsen and Huber (2025), and capital to GDP from BEA.

▷ back

### Capital Tax Gains

No capital gains in t=0. Budget constraint in t=1

$$c_1^j + q_1 \Delta_1^{j+} \leq w_1 h_1^j + d_1 heta_1^j + q_1 \Delta_1^{j-} - rac{ au}{2} (\Delta_1^{j-})^2 (q_1 - q_0)$$

Firms maximize

$$\sum_{j \in \{l,h\}} rac{ heta_1^j}{2} \left[ d_1 + (1 - au \Delta_1^{j-}) q_1 + au \Delta_1^{j-} q_0 
ight]$$

Households' Euler equation

$$q_0 = eta rac{u'(c_1^j)}{u'(c_0^j)} \left[ (d_1 + (1 - au \Delta_1^{j-}) q_1 + au \Delta_1^{j-} q_0 
ight], \qquad (1 - au \Delta_1^{j-}) q_1 + au \Delta_1^{j-} q_0 = eta rac{u'(c_2^j)}{u'(c_1^j)} d_2.$$

Then, the firm solves

$$V_0^F(k_0) = \max_{k_1, k_2 \geq 0} \sum_{i \in II, k_1} \frac{\theta_0^i}{2} \left| d_0 + \beta \frac{u'(c_1^j)}{u'(c_0^j)} d_1 + \beta^2 \frac{u'(c_2^j)}{u'(c_0^j)} d_2 \right|.$$

### Corporate discount rate wedge

Gormsen and Huber (2025) decompose the firm's discount factor Λ

$$\Lambda = \underbrace{r^{\mathit{fin}}}_{\mathsf{financial cost}} + \underbrace{\kappa}_{\mathsf{discount rate wedge}}$$

Model without commitment:

$$r^{fin} \equiv \log \left( rac{1}{ ilde{\delta}} 
ight) pprox r + \Phi^B, \quad ext{and} \quad \kappa \equiv \log \left( rac{1}{ ilde{eta}} 
ight) pprox \overline{\Phi} - \Phi^B.$$

Present bias generates the discount rate wedge

	Model	Data
Corporate discount rate wedge, percent	1.8	2.1

The model explains about 85% of the wedge

## Liquidity and the corporate discount rate wedge

More illiquid firms have higher wedges

$$\kappa_{it} = \alpha_t + \delta_i + \beta \text{ liquidity}_{i,t} + \gamma X_{i,t} + \varepsilon_{i,t}$$

Liquidity	0.228***	0.184***	0.230***	0.181***
	(0.016)	(0.012)	(0.016)	(0.012)
Observations	27163	27158	27163	27158
R-squared	0.266	0.668	0.266	0.669
FE .	Time	Firm, Time	Time	Firm, Time
Controls		·	Market cap	Market cap

Notes: Firm-quarter data,2002Q1 to 2021Q4. Standard errors (in parentheses) are clustered by firm. The left-hand side variable is in percent. Liquidity is measured with relative spreads from CRSP. The regressors are standardized, so that the coefficients estimate the impact of a 1 standard deviation increase.

- Iliquid firms have higher discount rate wedges
- Model suggests that present bias is a factor behind this empirical finding

### Empirics: More illiquid firms have higher discount rates

Relative spread	0.509***	0.281***	0.497***	0.278***
	(0.026)	(0.016)	(0.027)	(0.016)
Observations	27163	27158	27163	27158
R-squared	0.236	0.805	0.238	0.805
FE	Time	Firm, Time	Time	Firm, Time
Controls			Market cap	Market cap

Notes: The dataset is at the firm-quarter level and runs from 2002 to 2021. Standard errors (in parentheses) are clustered by firm. The left-hand side variable is in percent. The regressors are standardized, so that the coefficients estimate the impact of a 1 standard deviation increase. The specification includes fixed effects for time, or time and firm. Statistical significance is denoted by \*\*\* p < 0.01, \*\* p < 0.05, \* p < 0.1.

#### ▶ Back

### Corporate bonds

Firms can borrow at interest rate  $1+r^{cb}=\frac{1+r}{1-\tilde{\phi}}$  up to a limit

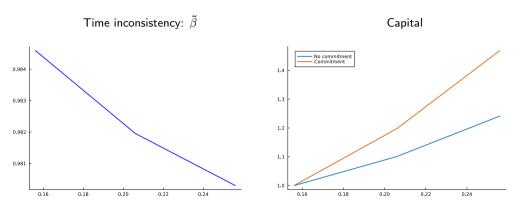
- $\blacktriangleright$  If  $\tilde{\phi}<\Phi^B$  the firm always borrows to the limit independently of its commitment.
- ▶ If  $\Phi^B < \tilde{\phi} < \overline{\Phi}$  only the firm without commitment borrows up to the limit.

### Implications:

- can alter financing but not investment and the time-inconsistency problem
- ▶ firms borrow even if bonds are more illiquid than stocks due to present bias
- rationalize corporate debt that does not rely on the tax advantage of debt

#### ▶ Back

### Demand of liquidity: increase idiosyncratic volatility



- Precautionary savings: more capital
- Time inconsistency: less capital
- ► → Larger increase in capital with commitment



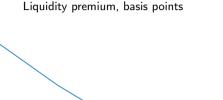
### Government bonds

- Introduce government bonds
- Lump-sum taxes to pay for the debt services
- Bonds market clearing

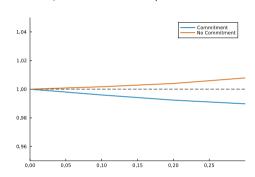
$$\int b'(\theta,b,h)d\Gamma(\theta,b,h)=B^{g}$$

ightharpoonup As  $B^g$  increases: more liquid assets

### Supply of liquidity & government bonds



#### Capital: Commitment / No commitment



Capital closer to complete markets

0.15

0.20

- lacktriangle Without commitment: less time inconsistency ightarrow more capital
- lacktriangle With commitment: less precautionary savings ightarrow less capital

0.25

▶ Back

36

34

32

30

28

0.00

0.05

0.10

#### Short-termism

#### Evidence on short-termism:

- ➤ an excessive focus on short-term results at the expense of long-term interests (Graham et al. 05, Terry 23, Fink 15)
- public firms distort their investment to meet short-term targets (Graham et al., 05).

Model: short-termism as a result of (i) trading frictions, and (ii) lack of commitment.

▶ Back

### Heterogeneous Firms: Public vs private firms

- Asker et al. (2015) finds that public firms invest substantially less than private firms.
- We add private firms to the benchmark equilibrium. Private firms are owned by only one household and are not traded in financial markets.
- The investment decisions of private firms are independent of  $\phi$ , while investment in public firms decreases with the transaction cost.
- For most values of  $\phi$  private firms invest more than public firms, consistent with the empirical evidence.

#### ▶ Back

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