

# The Cost of Capital and Misallocation in the United States

Miguel Faria-e-Castro  
FRB St. Louis

Julian Kozlowski  
FRB St. Louis

Jeremy Majerovitz  
University of Notre Dame

May 16th, 2025

Midwest Macroeconomics Meeting

Federal Reserve Bank of Kansas City

The views expressed herein are solely those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of St. Louis, the Board of Governors of the Federal Reserve, or the Federal Reserve System. These slides have been screened to ensure that no confidential bank or firm-level data have been revealed.

# The Cost of Capital and Misallocation in the United States

**Goal:** Measure how **dispersion in the cost of capital** affects the **allocation of capital**

## Methodological contribution:

- Adapt a standard **macrofinance model** to enable measurement using **micro data**
- Derive a **sufficient statistic** for misallocation using **credit registry data**

## Empirical Results (US):

- Low levels of misallocation in normal times ( $\approx 0.5\%$  of GDP)
- 🔥 Losses from misallocation increased to 1.1% of GDP in 2020-2021 🔥
- Driven by an increase in moral hazard and zombie lending 🧟

## Related Literature

- **Measuring Misallocation:**

- Seminal work by Restuccia and Rogerson (2008), Hsieh and Klenow (2009)
- **Contribution:** Use **cost of capital heterogeneity** as a proxy for **dispersion on the marginal product of capital**

- **Heterogeneity in Cost of Capital:**

- Developing countries: Banerjee and Duflo (2005), Cavalcanti, Kaboski, Martins, and Santos (2021)
- US: Gilchrist, Sim, and Zakrajek (2013), Gormsen and Huber (2023, 2024), Faria-e-Castro, Jordan-Wood, and Kozlowski (2024)
- **Contribution:**
  - Estimate firm-level cost of capital using **credit registry data**, correcting for default and LGD.
  - Derive and estimate **sufficient statistic** for misallocation

# Outline

1. Macrofinance model
2. Welfare & sufficient statistic for misallocation
3. Mapping to credit registry data
4. Empirical results in the US

## 1. Macrofinance Model

# Macrofinance Model

## Borrowers 🏠

- Produce output  $f(k, z)$
- Invest in capital  $k$
- Borrow long-term debt  $b$
- Limited liability: default

# Macrofinance Model

## Borrowers 🏠

- Produce output  $f(k, z)$
- Invest in capital  $k$
- Borrow long-term debt  $b$
- Limited liability: default

## Lenders 💰

- Discount rate  $\rho$
- Price loans competitively
- Recover  $\phi k$  in case of default

# Macrofinance Model

## Borrowers 🏠

- Produce output  $f(k, z)$
- Invest in capital  $k$
- Borrow long-term debt  $b$
- Limited liability: default

## Matching 🤝

- Borrower matched with lender
- Match efficiency:  $\rho$
- Heterogeneity in  $\rho$
- Wedge accounting:  $\rho = \bar{\rho} + \omega$

## Lenders 💰

- Discount rate  $\rho$
- Price loans competitively
- Recover  $\phi k$  in case of default



# Macrofinance Model

## Borrowers 🏠

- Produce output  $f(k, z)$
- Invest in capital  $k$
- Borrow long-term debt  $b$
- Limited liability: default

## Matching 🤝

- Borrower matched with lender
- Match efficiency:  $\rho$
- Heterogeneity in  $\rho$
- Wedge accounting:  $\rho = \bar{\rho} + \omega$

## Lenders 💰

- Discount rate  $\rho$
- Price loans competitively
- Recover  $\phi k$  in case of default

**Key question:** How do heterogeneity in  $\rho$  and financial frictions distort the allocation of capital?

## Firm's Problem

Value of Repayment:

$$V(k, b, z) = \max_{k', b'} \pi(k, b, z, k', b') + \beta \mathbb{E} \left[ \overbrace{\max \{ V(k', b', z'), 0 \}}^{\text{Limited liability}} \mid z \right]$$

Profits:

$$\pi(k, b, z, k', b') = f(k, z) + (1 - \delta)k - k' - \theta b + Q(k', b', z)(b' - (1 - \theta)b)$$

Price of Debt:

$$Q(k', b', z) = \frac{\mathbb{E}[\mathcal{P}(k', b', z')(\theta + (1 - \theta)Q(k'', b'', z')) + (1 - \mathcal{P}(k', b', z'))\phi k'/b' \mid k', b', z]}{\underbrace{1 + \rho}_{\text{borrower-lender efficiency}}}$$

## The Firm's Cost of Capital

Define the implicit interest rate paid by the firm as

$$1 + r_t^{firm} = \frac{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})]}{Q_t}$$

### Lemma 1 (Firm's Cost of Capital)

*The firm's cost of capital is:*

$$1 + r^{firm} = \frac{1 + \rho}{1 + \Lambda}$$

$$\Lambda \equiv \frac{\mathbb{E}_t [(1 - \mathcal{P}_{t+1}) \phi k' / b']}{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})]}$$

▷ *Proof*

$\Lambda$  is the **financial frictions' wedge** due limited liability and **partial recovery**  $\phi$

- $\phi = 0$ : no recovery after default, then  $r^{firm} = \rho$
- If  $\phi > 0$ , then  $\Lambda > 0$  and  $r^{firm} < \rho$ : **borrower only takes into account repayment states**

## Marginal Revenue Product of Capital

$$\underbrace{(1 + r_t^{firm})\mathcal{M}_t}_{\text{Cost of capital}} = \underbrace{\mathbb{E}_t[\mathcal{P}_{t+1}(f_k(k_{t+1}, z_{t+1}) + 1 - \delta)]}_{\text{Expected marginal revenue product of capital}} \quad (1)$$

where  $\mathcal{M}$  reflects the price feedback multiplier

$$\mathcal{M} \equiv \frac{1 - \gamma \times \frac{b'}{k'} \times \frac{\partial \log Q}{\partial \log k'}}{1 + \gamma \times \frac{\partial \log Q}{\partial \log b'}} \quad \gamma \equiv \frac{b' - (1 - \theta)b}{b'}$$

Wedge accounting:  $\omega$  is a wedge in the Euler equation

$$\left(1 + \frac{1 + \bar{\rho} + \omega}{1 + \Lambda}\right) \mathcal{M} = \mathbb{E}[MRPK]$$

Key empirical insight:

- Measure  $r_t^{firm}$  by measuring  $\rho$  and  $\Lambda$
- Intuition: heterogeneity in  $r_t^{firm} \rightarrow$  heterogeneity in MRPK

## 2. Welfare & Sufficient Statistic for Misallocation

# Aggregate Economy and Welfare

## Decentralized Equilibrium:

$$Y^{DE} + (1 - \delta)K^{DE} = \int_0^1 \mathbb{E}_t \left[ \mathcal{P}_{i,t+1}^{DE} \left( f(k_{i,t+1}^{DE}, z_{i,t+1}) + (1 - \delta)k_{i,t+1}^{DE} \right) + (1 - \mathcal{P}_{i,t+1}^{DE}) \cdot \phi k_{i,t+1}^{DE} \right] di$$

# Aggregate Economy and Welfare

## Decentralized Equilibrium:

$$Y^{DE} + (1 - \delta)K^{DE} = \int_0^1 \mathbb{E}_t \left[ \mathcal{P}_{i,t+1}^{DE} \left( f(k_{i,t+1}^{DE}, z_{i,t+1}) + (1 - \delta)k_{i,t+1}^{DE} \right) + (1 - \mathcal{P}_{i,t+1}^{DE}) \cdot \phi k_{i,t+1}^{DE} \right] di$$

## Planner's Problem:

- Takes as given  $\mathcal{P}^{DE}$  and aggregate capital  $K^{DE}$  (lower bound on full misallocation)
- Redistributes capital to maximize expected output

$$Y^* + (1 - \delta)K^{DE} = \max_{\{k_{i,t+1}^*\}_i} \int_0^1 \mathbb{E}_t \left[ \mathcal{P}_{i,t+1}^{DE} \left( f(k_{i,t+1}^*, z_{i,t+1}) + (1 - \delta)k_{i,t+1}^* \right) + (1 - \mathcal{P}_{i,t+1}^{DE}) \cdot \phi k_{i,t+1}^* \right] di$$
$$\text{s.t.} \quad \int_0^1 k_{i,t+1}^* di = K_{t+1}^{DE} \equiv \int_0^1 k_{i,t+1}^{DE} di$$

# Private vs Social Optimality

## Private optimality:

$$(1 + r_{i,t}^{firm}) \mathcal{M}_{i,t} = \mathbb{E}_t[\mathcal{P}_{i,t+1}^{DE}(f_k(k_{i,t+1}^{DE}, z_{i,t+1}) + 1 - \delta)]$$

## Planner's optimality:

- Define the **social marginal product of capital at firm  $i$** ,  $r_{i,t}^{social}$

$$1 + r_{i,t}^{social} \equiv \mathbb{E} \left[ \mathcal{P}_{i,t+1}^{DE} (f_k(k_{i,t+1}^*, z_{i,t+1}) + 1 - \delta) + \left(1 - \mathcal{P}_{i,t+1}^{DE}\right) \phi \right]$$

- Takes into account recovery in the case of default
- Optimality:** The planner wants to **equalize**  $r_{i,t}^{social}$  across firms



# Misallocation

## Proposition 1 (Misallocation)

Misallocation can be measured with  $\mathbb{E}[r^{social}]$  and  $\text{Var}(r^{social})$  as

$$\log(Y^*/Y^{DE}) \approx \frac{1}{2} \cdot \mathcal{E} \cdot \log\left(1 + \frac{\text{Var}(r^{social})}{(\mathbb{E}[r^{social}] + \delta)^2}\right)$$

▷ *Proof*

- This is an extension of Hughes and Majerovitz (2024) to a *dynamic economy with default*
- We can calibrate  $\mathcal{E} = \frac{1}{2}$  and  $\delta = 0.06$  ▷ [Calibration](#)
- **Misallocation can be measured with credit registry data:** We now show how to measure  $r_{i,t}^{social}$

### 3. Mapping to Credit Registry Data

## Data: FR Y-14Q (Schedule H.1)

- Quarterly loan-level panel on universe of loan facilities > \$1M
- Covers top 30/40 BHCs, 2014:Q4-2024Q4
- Detailed information on features of credit facilities
- Variables of interest: origination date, size, maturity, interest rate/spread, probability of default, loss given default, fixed vs. floating, type of loan
- Focus on term loans issued to non-government, non-financial US companies

## Asset Pricing of Term Loans

The **break-even** condition for a lender with discount rate  $\rho$  is

$$1 = \sum_{t=1}^T \left[ \frac{P^t \mathbb{E}_0[r_t] + P^{t-1}(1-P)(1-LGD)}{(1+\rho)^t} \right] + \frac{P^T}{(1+\rho)^T} \quad (2)$$

- Maturity  $T$
- Interest rate  $\mathbb{E}_0 r_t$  (fixed rate or fixed coupon over floating benchmark rate)
- Repayment probability  $P$  (constant over time)
- Loss given default  $LGD$  (constant over time)
- $\Rightarrow$  Solve for lender's discount rate:  $\rho$

▷ [Forward rates](#)

## Lender's Discount Rate

### Fixed contractual rate:

#### Lemma 2 (Lender's Discount Rate)

*For a fixed contractual rate loan:*

$$1 + \rho = P(1 + r) + (1 - P)(1 - LGD)$$

▷ *Proof*

Note:  $\rho$  independent of maturity  $T$  for fixed rate loans.

**Floating contractual rate:** Numerical solution of (2).

## Firm's Cost of Capital

### Lemma 3 (Firm's Cost of Capital)

We can solve for  $\Lambda$  as

$$\Lambda = \frac{(1 - P)(1 - LGD)}{1 + \rho - (1 - P)(1 - LGD)}$$
$$1 + r^{firm} = (1 + \rho) - \underbrace{(1 - P)(1 - LGD)}_{\text{Adjusted Default Probability}}$$

▷ Proof

- Adjusted Default Probability  $(1 - P)(1 - LGD)$ : a default event that does not result in a loss
- With probability  $(1 - P)(1 - LGD)$  the borrower defaults, but the lender receives payment
- Larger ADP → larger wedge between borrower and lender
- For fixed interest rate loans:  $1 + r^{firm} = (1 + r)P$

# Social Cost of Capital

## Lemma 4 (Social Cost of Capital)

*The social cost of capital is:*

$$\begin{aligned} 1 + r^{\text{social}} &= (1 + r^{\text{firm}})\mathcal{M} + (1 - P)(1 - LGD)lev \\ &= \underbrace{(1 + \rho)\mathcal{M}}_{\rho \text{ heterogeneity}} + \underbrace{(lev - \mathcal{M}) \cdot (1 - P) \cdot (1 - LGD)}_{\text{Adjusted default probability}} \\ &\quad \underbrace{\hspace{10em}}_{\text{Agency Friction}} \end{aligned}$$

- Social cost of capital = lender's discount rate + agency friction

## Sufficient Statistic for Misallocation

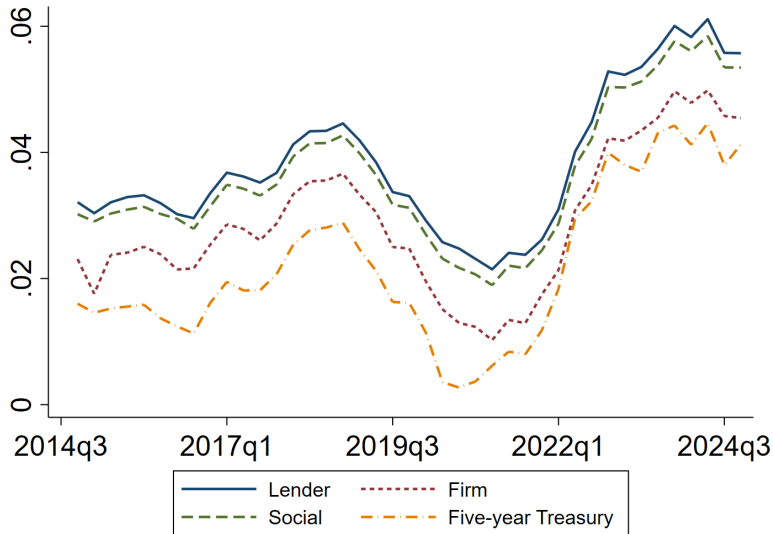
$$\log(Y^*/Y^{DE}) \approx \frac{1}{2} \cdot \mathcal{E} \cdot \log\left(1 + \frac{\text{Var}(r^{social})}{(\mathbb{E}[r^{social}] + \delta)^2}\right)$$
$$1 + r_i^{social} = (1 + \rho_i) \mathcal{M}_i + (lev_i - \mathcal{M}_i) \cdot (1 - P_i) \cdot (1 - LGD_i)$$

- Calibrate  $\mathcal{M} = 1$ . We also estimated  $\mathcal{M}$  and obtained similar results [▷ Estimate  \$\mathcal{M}\$](#)
- Can measure misallocation directly with credit registry data!
- Dispersion in  $r^{social}$  comes from:
  1. Dispersion in lender's discount rate,  $\rho$
  2. Dispersion in agency friction
  3. Covariance between  $\rho$  and agency friction



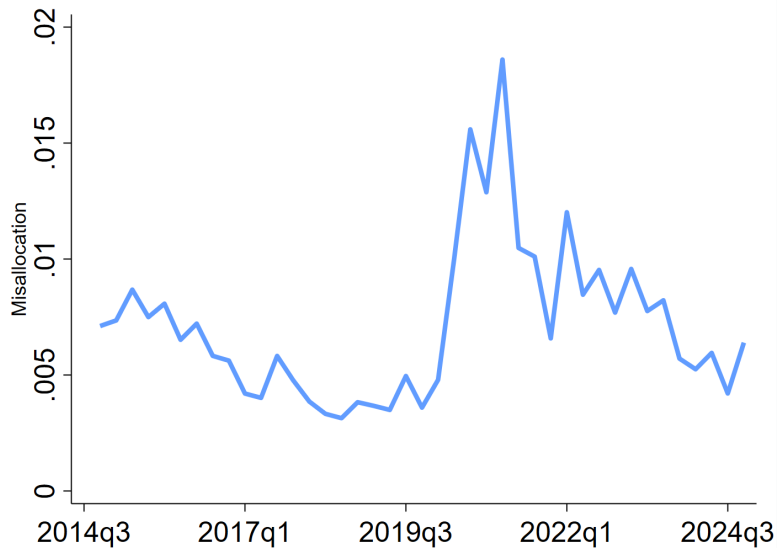
## 4. Empirical Results

## Average Discount Rate, Firm and Social Cost of Capital



- Rates follow the UST
- Social return exceeds firm cost: agency friction
- $\rho \approx r^{social} > r^{firm}$

## Misallocation



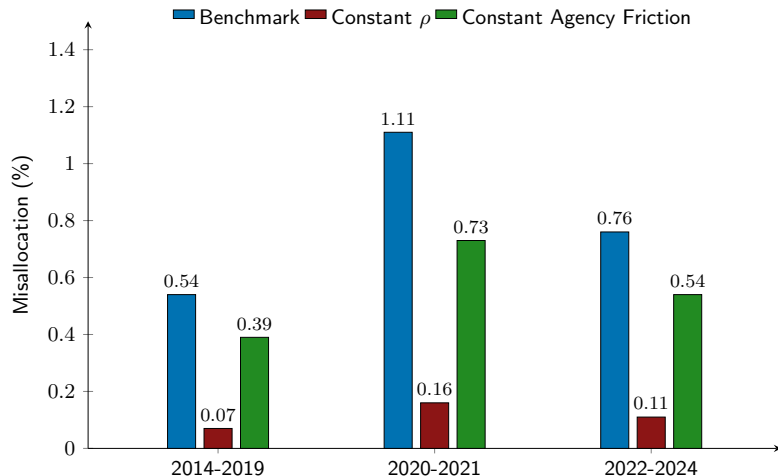
- About 0.5% before 2020
- Increase to 1.1% during 2020-2021
- Back to 0.8% in 2022-2024

## The 2020–2021 Increase in Misallocation

1. Predominantly explained by  $\rho$  heterogeneity, rather than agency frictions
2. Sharp rise in the coefficient of variation of  $\rho$
3. Dispersion in  $\rho$  is traced to changes in the distribution of contractual rates (not  $P$  or  $LGD$ )
4. The shift in contractual rates is driven by underpricing of very risky loans

⇒ Leads to moral hazard and zombie lending 🧟

## Sources of Misallocation: Heterogeneous $\rho$ vs. Agency Friction



- Mostly driven by heterogeneity in  $\rho$
- Interaction between  $\rho$  and agency friction  
( $0.54 > 0.07 + 0.39$ )
- The 2020-2021 spike is driven by an increase in the dispersion of  $\rho$

⇒ 1. Predominantly explained by  $\rho$  heterogeneity

## 2020-2021: Dispersion in $\rho$

- The heterogeneity in  $\rho$  is the most important driver of increase in misallocation during 2020-2021
- As rates decreased in 2020-2021, the mean  $\rho$  decreased too
- The standard deviation of  $\rho$  increased during this period

⇒ 2. Sharp rise in the coefficient of variation of  $\rho$

- We can approximate  $\rho \approx r - (1 - P)LGD$
- The coefficient of variation depends on: (i)  $r$ , (ii)  $(1 - P)LGD$  and (iii) the covariance

$$\frac{\mathbb{V}[\rho]^{0.5}}{\mathbb{E}[\rho]} \approx \frac{(\mathbb{V}[r] + \mathbb{V}[(1 - P)LGD] - 2\text{COV}[r, (1 - P)LGD])^{0.5}}{\mathbb{E}[r] - \mathbb{E}[(1 - P)LGD]}$$

⇒ 3. Dispersion in  $\rho$  is traced to changes in the distribution of contractual rates (not  $P$  or  $LGD$ )

## 2020-2021: Underpricing of Risky Loans

- Key observation: **underpricing** of **very risky loans**—offered with unusually favorable contractual rates
- These loans had **low implied**  $\rho$ , increasing overall dispersion

### Our hypothesis:

- Broad fiscal and monetary interventions (e.g., PPP, MSLP, PMCCF, SMCCF) supported distressed firms
- Lenders **inferred implicit government guarantees** for risky loans
- This created a **moral hazard** and **zombie lending**: Lenders took on more risk, expecting government bailouts in case of default

### Implication:

- Risk was mispriced, leading to **credit misallocation**
- **Absent guarantees**, risk would have been priced more accurately, improving allocative efficiency.



	Aleem 1990 Pakistan	Khwaja & Mian 2005 Pakistan	Cavalcanti et al. 2024 Brazil	Beraldi 2025 Mexico	This paper 2025 United States
Years of Data	1980–1981	1996–2002	2006–2016	2003–2022	2014–2024
Average contractual rate, %	78.7	14.1	83.0	16.8	3.9
St deviation of contractual rate, %	38.1	2.9	93.3	5.2	1.5
Default probability, %	2.7	16.9	4.0	8.9	1.4
Recovery rate (World Bank), %	42.8	42.8	18.2	63.9	81.0
Implied Misallocation, %	4.9	2.2	21.5	1.7	0.6

- **Developing countries** show higher mean and standard deviation of contractual rates
- **U.S.** shows lower mean and standard deviation of contractual rate, with **high recovery**
- **Brazil**: most extreme misallocation: 21.5%.
- Misallocation in the U.S. is small but non-trivial: **0.6%**.

# Conclusions

- Develop a macrofinance model with heterogeneous **costs of capital** and **limited liability**
- Derive a **new sufficient statistic** for measuring **capital misallocation** from credit registry data
- Map to U.S. credit registry data, estimating **firm-level cost of capital** and **social returns**
- 2020-2021 interventions lead to increased moral hazard and zombie lending 🧟 → **increased misallocation**

# Appendices

## Firm Interest Rate

$$\begin{aligned}\mathbb{E}_t \left[ \frac{\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})}{Q_t} \right] &= (1 + \rho) \frac{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})]}{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})] + \mathbb{E}_t [(1 - \mathcal{P}_{t+1}) \phi k' / b']} \\ &= (1 + \rho) \left( 1 + \frac{\mathbb{E}_t [(1 - \mathcal{P}_{t+1}) \phi k' / b']}{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})]} \right)^{-1} \\ &= (1 + \rho) (1 + \Lambda)^{-1}\end{aligned}$$

where

$$\Lambda \equiv \frac{\mathbb{E}_t [(1 - \mathcal{P}_{t+1}) \phi k' / b']}{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})]}$$

► Back

## Lender's Discount Rate: Fixed rate

$$1 = \sum_{t=1}^T \left( \frac{P}{1+\rho} \right)^t \left[ r + \frac{(1-P)}{P} (1-LGD) \right] + \left( \frac{P}{1+\rho} \right)^T$$

Let  $x = \frac{P}{1+\rho}$  so

$$1 = \left( r + \frac{(1-P)}{P} (1-LGD) \right) \frac{x}{1-x} (1-x^T) + x^T$$

Guess that  $1 + \rho = (1+r)P + (1-P)(1-LGD)$

$$\frac{1-x}{x} = \frac{1}{x} - 1 = \frac{(1+r)P + (1-P)(1-LGD)}{P} - 1 = r + \frac{1-P}{P} (1-LGD)$$

And, therefore

$$1 = 1 (1-x^T) + x^T$$

which validates the guess.

## Firm's Cost of Capital: Model:

$$Q_t = \frac{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1}) + (1 - \mathcal{P}_{t+1}) \phi k_{t+1} / b_{t+1}]}{1 + \rho}$$

Note that

$$Q_t = Q_t^P + Q_t^D$$

$$Q_t^P = \frac{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})]}{1 + \rho}$$

$$Q_t^D = \frac{\mathbb{E}_t [(1 - \mathcal{P}_{t+1}) \phi k_{t+1} / b_{t+1}]}{1 + \rho}$$

That is, we strip the bond into the payment in repay ( $Q_t^P$ ) and the payment in default ( $Q_t^D$ ).

We are interested in

$$\Lambda = \frac{\mathbb{E}_t [(1 - \mathcal{P}_{t+1}) \phi k_{t+1} / b_{t+1}]}{\mathbb{E}_t [\mathcal{P}_{t+1} (\theta + (1 - \theta) Q_{t+1})]}$$

$$\Lambda = \frac{Q_t^D}{Q_t^P}$$

## Firm's Cost of Capital: Measurement:

The firm defaults with probability  $(1 - P)$  and the lender recovers  $(1 - LGD)$ . Hence

$$Q_t^{D,data} = \frac{(1 - P)(1 - LGD)}{1 + \rho}$$

For the payment portion notice that at issuance we have the following condition

$$\begin{aligned} 1 &= \sum_{s=1}^T \left[ \frac{P^s \mathbb{E}_t[r_{t+s}] + P^{s-1} (1 - P)(1 - LGD)}{(1 + \rho)^s} \right] + \frac{P^T}{(1 + \rho)^T} \\ 1 &= \frac{(1 - P)(1 - LGD)}{1 + \rho} + P \frac{\mathbb{E}_t[r_{t+1}]}{1 + \rho} + \left( \sum_{s=2}^T \left[ \frac{P^s \mathbb{E}_t[r_{t+s}] + P^{s-1} (1 - P)(1 - LGD)}{(1 + \rho)^s} \right] + \frac{P^T}{(1 + \rho)^T} \right) \end{aligned}$$

So, we can define  $Q_t^{P,data}$  as  $1 = Q_t^{P,data} + Q_t^{D,data}$  so  $Q_t^{P,data} = 1 - Q_t^{D,data}$ . Finally

$$\Lambda^{data} = \frac{Q_t^{D,data}}{Q_t^{P,data}} = \frac{(1 - P)(1 - LGD)}{1 + \rho - (1 - P)(1 - LGD)}$$

## Forward Interest Rate Expectations

To estimate  $\rho$  for floating rate loans, we need estimates of  $\mathbb{E}_0[r_t]$

- Floating rate loans charge reference rate + spread
- We use smoothed daily yield curve estimates from the Fed Board, based on methodology in Gürkaynak, Sack, and Wright (2006)
- We assume expectations hypothesis: long rates reflect expected short rates
- Back out  $\mathbb{E}[r_t]$  for each loan, using treasury forward rate plus loan's spread
- *Note: Most loans actually use LIBOR/SOFR as the reference rate, not treasuries. For now we treat the rates as the same; they are very similar during our sample period.*



## Proof of Misallocation

- Formally, planner's problem is now the same as solving  $Y = \max_{\{k_i\}_i} \int_0^1 f_i(k_i) di$ , where  $f_i(k_i)$  is now expected output
- Apply Hughes and Majerovitz (2024), noting  $\frac{dY}{dk} = r^{social} + \delta$

$$\log(Y^*/Y^{DE}) \approx \frac{1}{2} \cdot \mathcal{E} \cdot \log\left(1 + \frac{\text{Var}(r^{social})}{(\mathbb{E}[r^{social}] + \delta)^2}\right)$$

- $\mathcal{E}$  is (negative) elasticity of output w.r.t. cost of capital ( $r^{social} + \delta$ )
- $\mathcal{E} = \frac{1}{2}$  corresponds to  $f(k, z) = z \cdot k^{1/3}$

## Calibration of $\mathcal{E}$

- $\mathcal{E}_i$  is the elasticity of expected output with respect to the cost of capital
- In a Cobb-Douglas setting, with  $f(k, z) = z \cdot k^\alpha$  and no default, the elasticity simplifies to  $\mathcal{E} = \frac{\alpha}{1-\alpha}$
- We calibrate  $\mathcal{E} = \frac{1}{2}$ , consistent with  $\alpha = \frac{1}{3}$ .

## Sources of Misallocation: Heterogeneous Cost of Capital and Agency Problem

**Counterfactual I:** What if all lenders have the same  $\bar{\rho}$ ?

$$1 + r_{social}^{cf,I} = \overline{(1 + \rho)\mathcal{M}} + (lev - \mathcal{M}) \cdot PD \cdot (1 - LGD)$$

Heterogeneity in  $r_{social}^{cf} \rightarrow$  Misallocation due to agency frictions

**Counterfactual II:** what if we equalize the agency friction?

$$1 + r_{social}^{cf,II} = (1 + \rho)\mathcal{M} + \overline{(lev - \mathcal{M}) \cdot PD \cdot (1 - LGD)}$$

Heterogeneity in  $r_{social}^{cf} \rightarrow$  Misallocation due to heterogeneous cost of capital

## Summary Statistics

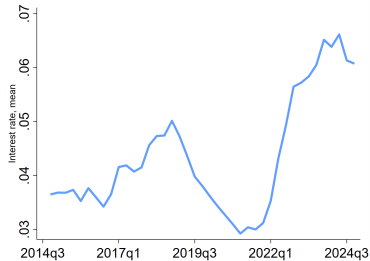
# Summary Statistics

Table: Summary Statistics

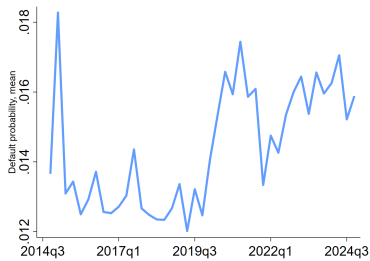
	mean	sd	p10	p50	p90
Interest rate	4.17	1.69	2.21	3.93	6.59
Maturity (yrs)	6.85	4.64	3.00	5.00	10.25
$\rho$ (%)	3.75	1.69	2.05	3.69	5.88
$r^{firm}$ (%)	2.82	2.75	0.87	3.04	5.26
$r^{social}$ (%)	3.54	1.88	1.77	3.53	5.71
Prob. Default (%)	1.42	2.37	0.19	0.82	2.85
LGD (%)	34.50	13.20	16.00	36.00	50.00
Loan amount (M)	10.77	68.81	1.11	2.55	22.64
Sales (M)	1,254.73	5,923.53	2.17	58.80	1,556.58
Assets (M)	1,770.83	8,956.78	1.06	35.52	1,792.00
Leverage (%)	72.03	24.57	42.57	71.17	100.00
Return on assets (%)	22.61	29.05	4.68	15.56	44.04
N Loans	62687				
N Firms	38587				
N Fixed Rate	31540				
N Variable Rate	31147				

## Raw Data: Contractual Rate, Default, LGD

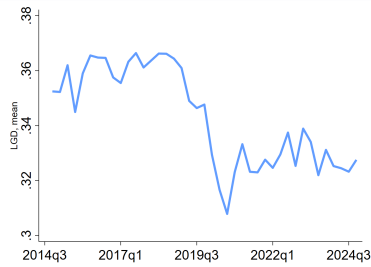
Contractual rate (fixed only)



Default Probability



LGD



- 2020-2021: Increase in default probability
- Modest decline in losses given default (better recovery)

## Variance decomposition

## Variance decomposition

- Decompose total variance in: time, firm, bank, and error
- Keep firms with 5 or more securities



## Variance Decomposition

	Time	Bank	Firm	Loan
Interest rate	71.88	1.63	13.45	13.04
$\rho$	61.94	3.08	14.02	20.96
$r^{firm}$	33.23	4.25	20.12	42.4
$r^{social}$	53.84	3.87	16.21	26.08
N Firms	1681			
N Securities	14738			

**Table:** Variance decomposition of interest rates and cost of capital ( $\rho$ ,  $r^{firm}$ , and  $r^{social}$ )

## Data Cleaning and Sample Construction

**Sample period:** We use FR Y-14Q Schedule H.1 data from 2014Q4 onward, due to improvements in reporting consistency and data quality.

### Borrower Filters:

- Drop loans without a Tax ID
- Keep only Commercial & Industrial loans to nonfinancial U.S. addresses
- Drop borrowers with NAICS codes:
  - 52 (Finance and Insurance), 92 (Public Administration)
  - 5312 (Real Estate Agents), 551111 (Bank Holding Companies)

# Data Cleaning and Sample Construction

## Loan Filters:

- Drop loans with:
  - Negative committed exposure
  - Utilized exposure exceeding committed exposure
  - Origination after or maturity before report date
- Keep only “vanilla” term loans (Facility type = 7)
- Drop loans with:
  - Mixed-rate structures
  - Maturity outside 110 years
  - Implausible interest rates or spreads (outside 1st99th percentile, or  $> 50\%$ )
  - Missing or invalid PD/LGD values (outside  $[0, 1]$ )
  - $PD = 1$  (flagged as in default)

## Estimating $\mathcal{M}$

$$\mathcal{M}_t = \frac{1 - \gamma \times \frac{Qb'}{k'} \times \frac{\partial \log Q}{\partial \log k'}}{1 + \gamma \times \frac{\partial \log Q}{\partial \log b'}}$$

Given estimates for the function  $Q$ ,  $\gamma$ , and firm leverage  $Qb'/k'$  we can compute  $\mathcal{M}$

### Compute $Q$

- Compute  $Q$  for every loan origination
- Loans are modeled as perpetuities that decay at a geometric rate  $\theta$ , we can write  $Q$  as the present value of all future payments, discounted at the contractual interest rate  $r$ :

$$Q = \frac{\theta + (1 - \theta)Q}{1 + r} = \frac{\theta}{r + \theta}$$

$r$  is directly observed in the data, and we can apply the common approximation that  $\theta = 1/T$

## Estimating $\mathcal{M}$ : $Q$ elasticities

- We approximate (the log of)  $Q$  as a polynomial of investment, borrowing, productivity and  $\rho$
- Investment: tangible assets
- Borrowing: total debt owed by the firm at loan origination
- Productivity: sales over tangible assets (a measure of TFPR following Hsieh Klenow 2009)
- We estimate:

$$\begin{aligned}\log Q_i = & \alpha + \beta_k \log k_i + \beta_b \log b_i + \beta_z \log z_i + \beta_\rho \rho_i \\ & + \beta_{k,k} (\log k_i)^2 + \beta_{k,b} \log k_i \times \log b_i + \beta_{k,z} \log k_i \times \log z_i + \beta_{k,\rho} \log k_i \times \rho_i \\ & + \beta_{b,b} (\log b_i)^2 + \beta_{b,z} \log b_i \times \log z_i + \beta_{b,\rho} \log b_i \times \rho_i \\ & + \beta_{z,z} (\log z_i)^2 + \beta_{z,\rho} \log z_i \times \rho_i + \beta_{\rho,\rho} (\rho_i)^2 + \epsilon_i\end{aligned}$$

- Compute the partial derivatives of  $\log Q$  with respect to investment and borrowing.

## Estimating $\mathcal{M}$ : Results

- The distribution is extremely concentrated around 1.
- The mean is equal to 0.996 and the median to 0.997, with a standard deviation of 0.006.
- The two measures of misallocation are extremely similar
- Taken together, these results suggest that our assumption that  $\mathcal{M} = 1$  is a good one.

## Approximating Misallocation

Assumptions:

- Recovery rates from the World Banks Doing Business
- We use the lender's cost of capital,  $\rho$ , in place of the social cost of capital,  $r_{social}$  (don't have leverage)
- Use the fixed rate formula for  $\rho$  and assume that the probability of default and the losses given default do not vary across firms
- This allows us to compute a cost of misallocation.
- The cost of misallocation for the US is similar to the actual cost

## References I

- [ ] Abhijit V. Banerjee and Esther Duflo. Chapter 7 growth theory through the lens of development economics. In Handbook of Economic Growth, pages 473–552. Elsevier, 2005. doi: 10.1016/s1574-0684(05)01007-5.
- [ ] Tiago Cavalcanti, Joseph Kaboski, Bruno Martins, and Cezar Santos. Dispersion in Financing Costs and Development. April 2021. doi: 10.3386/w28635.
- [ ] Miguel Faria-e-Castro, Samuel Jordan-Wood, and Julian Kozlowski. An Empirical Analysis of the Cost of Borrowing. Working Papers 2024-016, Federal Reserve Bank of St. Louis, July 2024. URL <https://ideas.repec.org/p/fip/fedlwp/98542.html>.
- [ ] Simon Gilchrist, Jae W. Sim, and Egon Zakrajek. Misallocation and financial market frictions: Some direct evidence from the dispersion in borrowing costs. Review of Economic Dynamics, 16(1):159–176, January 2013. ISSN 1094-2025. doi: 10.1016/j.red.2012.11.001.
- [ ] Niels Joachim Gormsen and Kilian Huber. Corporate Discount Rates. June 2023. doi: 10.3386/w31329.
- [ ] Niels Joachim Gormsen and Kilian Huber. Firms Perceived Cost of Capital. June 2024. doi: 10.3386/w32611.
- [ ] Chang-Tai Hsieh and Peter J. Klenow. Misallocation and manufacturing tfp in china and india. Quarterly Journal of Economics, 124(4):1403–1448, November 2009. doi: 10.1162/qjec.2009.124.4.1403.
- [ ] Diego Restuccia and Richard Rogerson. Policy distortions and aggregate productivity with heterogeneous establishments. Review of Economic Dynamics, 11(4):707–720, October 2008. doi: 10.1016/j.red.2008.05.002.