The Cost of Capital and Misallocation in the United States

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June 20 2025

BSE Summer Forum

Macro Fluctuations with Micro Frictions

The Cost of Capital and Misallocation in the United States

Goal: measure how dispersion in the cost of capital affects its allocation

Methodological contribution:

- Adapt a standard dynamic corporate finance model to enable measurement using micro data
- Derive a sufficient statistic for misallocation using credit registry data

Empirical Results (US):

- Low levels of misallocation in normal times ($\approx 0.5\%$ of GDP)
- Losses from misallocation increased to 1.1% of GDP in 2020-2021
- Possibly tied to mispricing of credit due to credit market interventions

Related literature

- **Measuring misallocation:**
  - Seminal work by Restuccia and Rogerson (2008), Hsieh and Klenow (2009)
  - Contribution: use heterogeneity in funding costs to measure dispersion in MPK
- **Heterogeneity in the cost of capital:**
  - Developing countries: Banerjee and Duflo (2005), Cavalcanti, Kaboski, Martins, and Santos (2024)
  - US: Gilchrist, Sim, and Zakrajsek (2013), David, Schmid, and Zeke (2022), Gormsen and Huber (2023, 2024), Faria-e-Castro, Jordan-Wood, and Kozlowski (2024)
  - Contribution:
    - Estimate firm cost of capital using credit registry data, correcting for maturity, default, etc.
    - Derive and estimate sufficient statistic for misallocation

Outline

1. Model
2. Welfare and misallocation
3. Measurement with credit registry data
4. Empirical results for the US

1. Model

Model

Borrowers

- Produce output $f(k_i, z_i)$
- Invest in capital k_i
- Long-term debt b_i
- Limited liability

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Lenders 💰

- Discount rate ρ_i
- Competitive pricing
- Recover $\phi_i k_i$ in default

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Matching 🤝

- Borrower-lender match
- $\rho_i \sim$ match efficiency
- Heterogeneity in ρ_i

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Key question: how do heterogeneity in ρ_i and financial frictions distort the allocation of capital?

Firm's problem

Value of repayment:

$$V_i\left(k_i, b_i, z_i\right) = \max_{k_i', b_i'} \; \pi_i\left(k_i, b_i, z_i, k_i', b_i'\right) + \beta \mathbb{E} \overbrace{\left[\max\left\{V_i\left(k_i', b_i', z_i'\right), 0\right\} \middle| z_i\right]}^{\text{Limited liability}}$$

Profits:

$$\pi_i\left(k_i, b_i, z_i, k_i', b_i'\right) = f\left(k_i, z_i\right) + \left(1-\delta\right) k_i - k_i' - \theta b_i + Q_i\left(k_i', b_i', z_i\right)\left(b_i' - \left(1-\theta_i\right) b_i\right)$$

Price of debt:

$$Q_i\left(k_i', b_i', z_i\right) = \frac{\mathbb{E}\left[\mathcal{P}_i\left(k_i', b_i', z_i'\right)\left(\theta_i + \left(1-\theta_i\right) Q_i\left(k_i'', b_i'', z_i'\right)\right) + \left(1 - \mathcal{P}_i\left(k_i', b_i', z_i'\right)\right) \frac{\phi_i k_i'}{b_i'} \middle| k_i', b_i', z_i\right]}{\underbrace{1+\rho_i}_{\text{lender discount rate / match efficiency}}}$$

Firm's cost of capital

Define the implicit interest rate paid by the firm as

$$1 + r_i^{firm} = \frac{\mathbb{E}\left[\mathcal{P}_i'(\theta_i + (1-\theta_i)Q_i') \middle| k_i', b_i', z_i\right]}{Q_i}$$

Lemma 1 (Firm cost of capital)

The firm cost of capital is:

$$1 + r_i^{firm} = \frac{1+\rho_i}{1+\Lambda_i} \qquad \Lambda_i := \frac{\mathbb{E}\left[(1-\mathcal{P}_i')\,\phi_i(k_i')/b_i' \middle| k_i', b_i', z_i\right]}{\mathbb{E}\left[\mathcal{P}_i'\,(\theta + (1-\theta_i)Q_i') \middle| k_i', b_i', z_i\right]}$$

▷ *Proof*

Λ_i: financial frictions wedge that arises due to limited liability and partial recovery ϕ_i

- $\phi_i = 0$: no recovery after default, then $r_i^{firm} = \rho_i$
- If $\phi_i > 0$, then $\Lambda_i > 0$ and $r_i^{firm} < \rho_i$: borrower only takes into account repayment states

Marginal revenue product of capital (MRPK)

$$\underbrace{(1 + r_i^{firm})\mathcal{M}_i}_{\text{cost of capital}} = \underbrace{\mathbb{E}[\mathcal{P}_i'(f_k(k_i', z_i') + 1 - \delta) |\, k_i', b_i', z_i]}_{\text{expected marginal revenue product of capital}} \tag{1}$$

where $\mathcal{M}_i$ captures the *price impact* of the firm's actions

$$\mathcal{M}_i := \frac{1 - \gamma_i \times \frac{Q_i \cdot b_i'}{k_i'} \times \frac{\partial \log Q_i}{\partial \log k_i'}}{1 + \gamma_i \times \frac{\partial \log Q_i}{\partial \log b_i'}}, \qquad \gamma_i := \frac{b_i' - (1 - \theta_i) b_i}{b_i'}$$

- Heterogeneity in r_i^{firm} $\rightarrow$ heterogeneity in $MRPK_i$
- Approach: measure r_i^{firm} by measuring ρ_i and Λ_i

2. Welfare and misallocation

Aggregate economy and welfare

Decentralized Equilibrium:

$$Y^{DE} + (1-\delta)K^{DE} = \int_0^1 \mathbb{E}_t \left[\mathcal{P}^{DE}_{i,t+1}\left(f(k^{DE}_{i,t+1}, z_{i,t+1}) + (1-\delta)k^{DE}_{i,t+1}\right) + (1-\mathcal{P}^{DE}_{i,t+1}) \cdot \phi k^{DE}_{i,t+1}\right] \mathrm{d}i$$

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Planner's problem:

- Inner problem: redistribute $\{k_{i,t+1}\}_i$ taking exit decisions and K^{DE} as given ▷ full planner problem
- Lower bound on full misallocation:

$$\max_{\left\{k^*_{i,t+1}\right\}_i} \quad \int_0^1 \mathbb{E}_t \left[\mathcal{P}^{DE}_{i,t+1}\left(f(k^*_{i,t+1}, z_{i,t+1}) + (1-\delta)k^*_{i,t+1}\right) + (1-\mathcal{P}^{DE}_{i,t+1})\cdot \phi k^*_{i,t+1}\right] \mathrm{d}i$$

$$\text{s.t.} \quad \int_0^1 k^*_{i,t+1}\mathrm{d}i = K^{DE}_{t+1}$$

Private vs. social optimality

Private optimality:

$$(1 + r_{i,t}^{firm})\mathcal{M}_{i,t} = \mathbb{E}_t[\mathcal{P}_{i,t+1}^{DE}(f_k(k_{i,t+1}^{DE}, z_{i,t+1}) + 1 - \delta)]$$

Planner optimality:

- Define the social marginal product of capital at firm i, $r_{i,t}^{social}$

$$1 + r_{i,t}^{social} \equiv \mathbb{E}\left[\mathcal{P}_{i,t+1}^{DE}\left(f_k\left(k_{i,t+1}, z_{i,t+1}\right) + 1 - \delta\right) + \left(1 - \mathcal{P}_{i,t+1}^{DE}\right)\phi\right]$$

- Takes into account recovery in case of default

- Optimality: planner **equalizes** $r_{i,t}^{social}$ across firms at $\{k_{i,t+1}^{*}\}_i$

Misallocation

Proposition 1 (Misallocation)

Misallocation can be measured with $\mathbb{E}\left[r_i^{social}\right]$ *and* $Var\left(r_i^{social}\right)$ *as*

$$\log\left(Y^*/Y^{DE}\right) \approx \frac{1}{2}\cdot\mathcal{E}\cdot\log\left(1+\frac{Var\left(r_i^{social}\right)}{\left(\mathbb{E}\left[r_i^{social}\right]+\delta\right)^2}\right)$$

▷ *Proof*

- Extend Hughes and Majerovitz (2025) to a dynamic economy with default
- Set $\mathcal{E} = \frac{1}{2}$ and $\delta = 0.06$ ▷ calibration
- **Next:** we show how to measure r_i^{social} using credit registry data

3. Measurement with credit registry data

Data: FR Y-14Q (Schedule H.1)

▷ summary stats. ▷ time series

- Quarterly loan-level panel on universe of loan facilities $> \$1M$
- Covers top 30/40 BHCs, 2014:Q4-2024Q4
- Detailed information on features of credit facilities
  - Origination date, size, maturity, interest rate/spread, probability of default, loss given default, fixed vs. floating, type of loan, etc.
- Focus on term loans issued to non-government, non-financial US companies
- Cannot consider credit lines due to lack of information on fees.

Data: Summary Statistics

	Mean	Median	sd
Contractual rate (%)	4.17	3.93	1.69
Maturity (yrs)	6.85	5.00	4.64
Prob. Default (%)	1.42	0.82	2.37
LGD (%)	34.50	36.00	13.20
Loan amount (M)	10.77	2.55	68.81
Assets (M)	1,770.83	35.52	8,956.78
Leverage (%)	72.03	71.17	24.57
Return on assets (%)	22.61	15.56	29.05
2014:Q4-2024Q4. 38,587 firms, 62,687 loans.			

Pricing term loans

The break-even condition for a lender with discount rate ρ_i is

$$1 = \sum_{t=1}^{T_i} \left[\frac{P_i^t \mathbb{E}_0 \left[r_{i,t} \right] + P_i^{t-1} (1 - P_i) \left(1 - LGD_i \right)}{\left(1 + \rho_i \right)^{t}} \right] + \frac{P_i^{T_i}}{\left(1 + \rho_i \right)^{T_i}} \tag{2}$$

- T_i: maturity
- $\mathbb{E}_0[r_{i,t}]$: fixed interest rate or fixed spread over floating benchmark rate ▷ forward rates
- P_i: repayment probability (constant over time)
- LGD_i: loss given default (constant over time)
- $\Rightarrow$ Solve for lender's discount rate: ρ_i

Lender's discount rate

Fixed contractual rate:

Lemma 2 (Lender's discount rate)

For a fixed contractual rate loan:

$$1 + \rho_i = P_i \left(1 + r_i\right) + \left(1 - P_i\right)\left(1 - LGD_i\right)$$

▷ *Proof*

- ρ_i is independent of maturity T_i for fixed rate loans
- **Floating rate:** numerical solution of (2)

Firm cost of capital

Lemma 3 (Firm cost of capital)

We can solve for Λ_i as

$$\Lambda_i = \frac{(1 - P_i)(1 - LGD_i)}{1 + \rho_i - (1 - P_i)(1 - LGD_i)}$$

and write the firm cost of capital as

$$1 + r_i^{firm} = (1 + \rho_i) - (1 - P_i)(1 - LGD_i)$$

▷ *Proof*

- $(1 - P_i)(1 - LGD_i) \simeq$ prob. of default event that does not result in a loss for the lender
- Measures pricing wedge between borrower and lender
- For fixed interest rate loans, use $(1 + \rho_i)$ as in Lemma 2 to write $1 + r_i^{firm} = (1 + r_i) P_i$

Social cost of capital

Lemma 4 (Social cost of capital)

The social cost of capital can be written as:

$$
\begin{aligned}
1 + r_i^{social} &= (1 + r_i^{firm})\mathcal{M}_i + (1 - P_i)(1 - LGD_i)lev_i \\
&= \underbrace{(1 + \rho_i)\,\mathcal{M}_i}_{\text{lender discount rate}} + \underbrace{(lev_i - \mathcal{M}_i)\cdot(1 - P_i)\cdot(1 - LGD_i)}_{\text{wedge due to financial frictions}}
\end{aligned}
$$

- social cost of capital $\simeq$ lender discount rate + wedge due to financial frictions
- Wedge due to financial frictions:
  - **Lenders** care about average recovery per dollar of debt: $\phi_i(k_i)/b_i = \mathcal{M}_i(1 - LGD_i)$
  - **Planner** cares about the marginal recovery: $\phi_i'(k_i) = (1 - LGD_i) \times lev_i$
  - Coincide when $lev_i = \mathcal{M}_i$

Sufficient statistic for misallocation

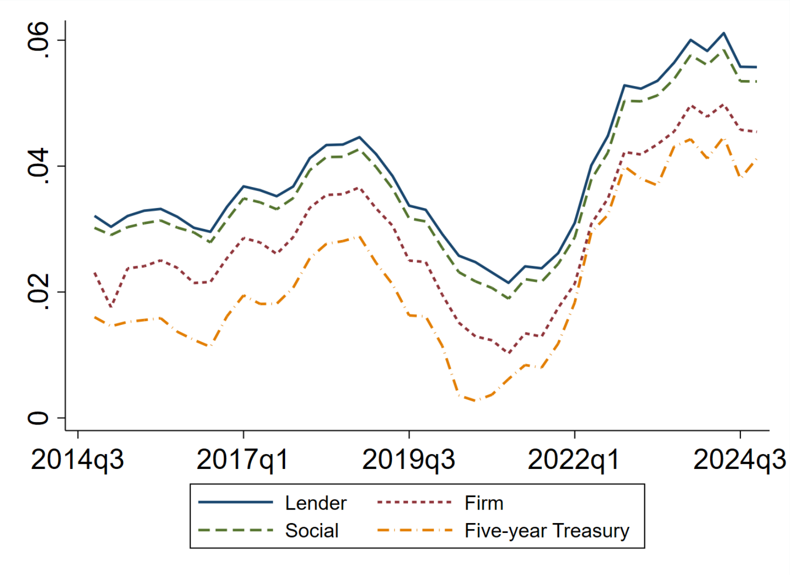
$$\log\left(Y^*/Y^{DE}\right) \approx \frac{1}{2}\cdot\mathcal{E}\cdot\log\left(1+\frac{\text{Var}\left(r_i^{social}\right)}{\left(\mathbb{E}\left[r_i^{social}\right]+\delta\right)^2}\right)$$

$$1+r_i^{social} = (1+\rho_i)\,\mathcal{M}_i + (lev_i - \mathcal{M}_i)\cdot(1-P_i)\cdot(1-LGD_i)$$

- Set $\mathcal{M}_i = 1$; reasonable approximation given our model ▷ Estimate $\mathcal{M}$
- Can measure misallocation directly with credit registry data!
- Dispersion in r_i^{social} comes from:
  1. Dispersion in lender's discount rate, ρ_i
  2. Dispersion in financial frictions wedge
  3. Covariance between ρ_i and financial frictions wedge

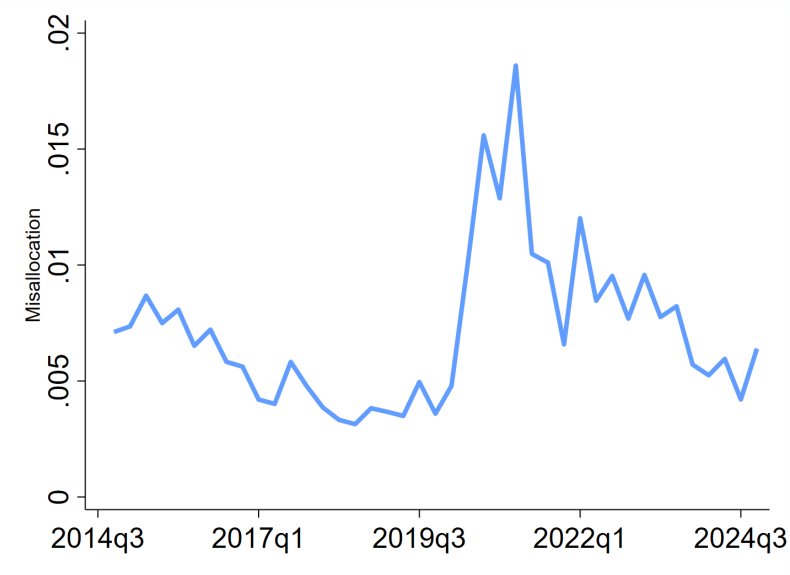
4. Empirical results

Average Discount Rate, Firm and Social Cost of Capital



- Rates follow 5y UST
- Financial frictions:
  $\mathbb{E}\left[r_i^{social}\right] > \mathbb{E}\left[r_i^{firm}\right]$
- $\mathbb{E}\left[r_i^{social}\right] \approx \mathbb{E}\left[\rho_i\right]$

Misallocation in the US, 2014-2024



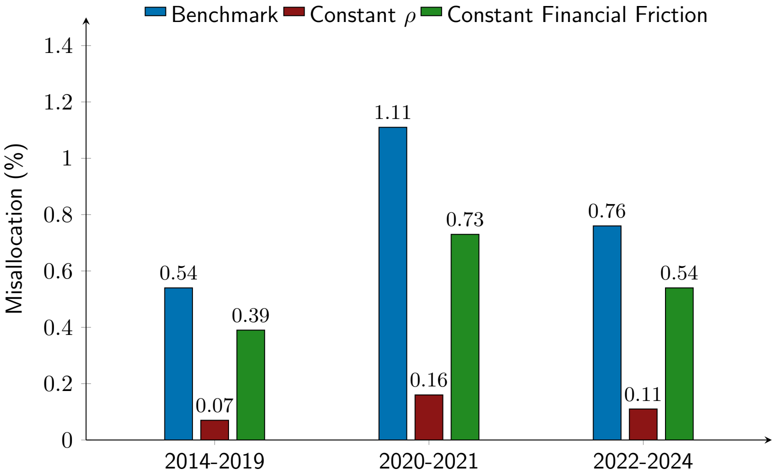
- About 0.5% before 2020
- ↑ to 1.1% in 2020-2021
- ↓ to 0.8% in 2022-2024

The 2020–2021 increase in misallocation

1. Predominantly explained by dispersion in ρ_i, rather than financial frictions wedge
2. Sharp rise in the coefficient of variation of ρ_i
3. Dispersion in ρ_i is traced to changes in the distribution of contractual rates (not P_i or LGD_i)
4. Driven by underpricing of very risky loans

1. The 2020-21 increase: sources of misallocation

▷ Decomposition

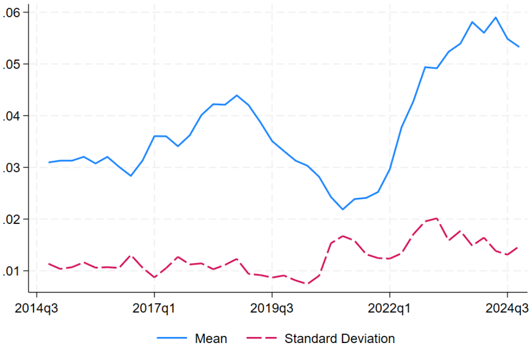


- Mostly driven by heterogeneity in ρ_i
- Interaction between ρ_i and financial frictions $(0.54 > 0.07 + 0.39)$

$\Rightarrow$ 1. Predominantly explained by dispersion in ρ_i

2. The 2020-21 increase: dispersion in ρ_i

Heterogeneity in ρ_i is the most important driver of increase in misallocation during 2020-21



- As policy rates decreased in 2020-21, so did the mean ρ_i
- The standard deviation of ρ_i increased during this period

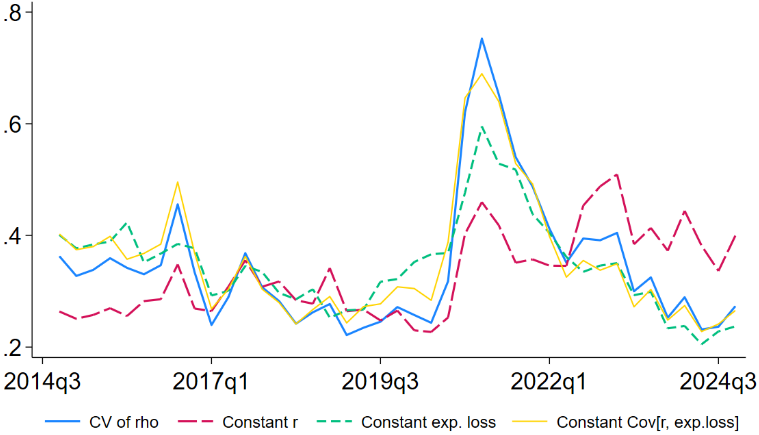
$\Rightarrow$ 2. Sharp rise in the coefficient of variation of ρ_i

3. The 2020-21 increase: role of contractual rates

- Approximate $\rho_i \approx r_i - (1 - P_i)LGD_i$
- The coefficient of variation depends on: (i) r_i, (ii) $(1 - P_i)LGD_i$ and (iii) their covariance

$$\frac{\mathbb{V}[\rho_i]^{0.5}}{\mathbb{E}[\rho_i]} \approx \frac{\left(\mathbb{V}[r_i] + \mathbb{V}[(1 - P_i)LGD_i] - 2\,\mathbb{COV}[r_i, (1 - P_i)LGD_i]\right)^{0.5}}{\mathbb{E}[r_i] - \mathbb{E}[(1 - P_i)LGD_i]}$$

3. Decomposition of the coefficient of variation of ρ_i



⇒ 3.Dispersion in ρ_i is traced to changes in the distribution of contractual rates r_i (not P_i or LGD_i)

4. The 2020-21 increase: underpricing of risky loans

- Very risky loans—offered with unusually favorable contractual rates
- These loans have low implied ρ_i, increasing overall dispersion

Our hypothesis:

- Broad fiscal and monetary interventions (PPP, MSLP, PMCCF, SMCCF) supported distressed firms
- Lenders inferred explicit and implicit government guarantees for risky loans
- Moral hazard/zombie lending

Implication:

- Risk was mispriced, leading to credit misallocation
- Absent guarantees, risk would have been priced more accurately, improving allocative efficiency.

Risk Premia & Aggregate Shocks

- Alternative hypothesis: Rise in ρ reflects higher **risk premia** as lenders demand extra compensation amid extreme uncertainty (e.g. COVID-19).
- Firms differ in exposure to aggregate shocks $\Rightarrow$ heterogeneous risk premia need not imply misallocation (David et al., 2022).
- Our framework is steady-state $\Rightarrow$ cannot model time-varying aggregate shocks or risk-premium spikes.
- Data contradict the risk-premia story:
  - Average ρ **falls** from 3.6% (2014-19) to 2.7% (2020-21).
  - Skewness becomes **more negative**: $-2.6 \rightarrow -3.5$ (left tail thickens).
- Interpretation: Risk premia likely **declined**, perhaps owing to explicit/implicit policy guarantees.

Cross-country comparison

▷ Assumptions

	Aleem 1990 Pakistan	Khwaja & Mian 2005 Pakistan	Cavalcanti et al. 2024 Brazil	Beraldi 2025 Mexico	This paper 2025 United States
Years of data	1980–1981	1996–2002	2006–2016	2003–2022	2014–2024
$\mu(r_i)$, %	78.7	14.1	83.0	16.8	3.9
$\sigma(r_i)$, %	38.1	2.9	93.3	5.2	1.5
$\mu(1-P_i)$, %	2.7	16.9	4.0	8.9	1.4
$\mu(1-LGD_i)$, % (World Bank)	42.8	42.8	18.2	63.9	81.0
Implied misallocation, %	4.9	2.2	21.5	1.7	0.6

- Developing countries: higher mean and standard deviation of contractual rates
- U.S.: lower mean and standard deviation of contractual rates, higher recovery
- Brazil: most extreme misallocation: 21.5%.
- Misallocation in the U.S. small but non-trivial: 0.6%.

Conclusions

- Develop a framework to measure misallocation using credit registry data
  1. Standard macrofinance model as measurement device
  2. Sufficient statistic for capital misallocation
  3. Inputs: standard credit registry variables (r, P, LGD, T, etc.)
- Application to U.S. credit registry data (FR Y-14Q)
  1. Estimate lender discount rates, firm-level cost of capital and social cost of capital
  2. Misallocation around 0.5% in normal times
  3. Sharp rise in 2020-21, possible tied to credit market interventions

Appendices

Proof: firm cost of capital

▷ back

$$\mathbb{E}_t\left[\frac{\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)}{Q_t}\right]=(1+\rho)\,\frac{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]+\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k'/b'\right]}$$

$$=(1+\rho)\left(1+\frac{\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k'/b'\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]}\right)^{-1}$$

$$=(1+\rho)\,(1+\Lambda)^{-1}$$

where

$$\Lambda\equiv\frac{\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k'/b'\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]}$$

Full planner problem

▷ back

$$U^* = \max_{\{\{k_{i,t}(S^{t-1}),\omega_{i,t}(S^t)\}_i\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \cdot u\left(Y_t - I_t\right)$$

$$\text{s.t.} \quad \omega_{i,t}\left(S^t\right) \in \{0,1\}\, \forall i$$

$$\omega_{i,t+1}\left(S^{t+1}\right) \geq \omega_{i,t}\left(S^t\right)\, \forall S^t \subset S^{t+1}, \forall i$$

Can separate into outer (dynamic) and inner (static) problems:

$$U^* = \max_{\left\{K_t,\{\omega_{i,t}(S^t)\}_{i\in[0,1]}\right\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \cdot u\left(\left(\max_{\left\{\{k_{i,t}(S^{t-1})\}_{i\in[0,1]}\right\}_{t=1}^{\infty}} Y_t\right) - I_t\right)$$

Rewrite inner problem as:

$$Y_t^*\left(K_t, \{\omega_{it}\}_{i\in[0,1]}\right) = \max_{\{k_{i,t}\}_{i\in[0,1]}} \int_0^1 \mathbb{E}_{t-1}\left[\omega_{it} \cdot f\left(k_{it}; z_{it}\right) - (1-\omega_{it}) \cdot \left((1-\delta)\, k_{it} - \phi\left(k_{it}\right)\right)\right] di$$

$$\text{s.t.} \quad K_t = \int_0^1 k_{it}\, di$$

Proof of Misallocation

▷ Back

- Formally, planner's problem is now the same as solving $Y = \max_{\{k_i\}_i} \int_0^1 f_i(k_i) \mathrm{d}i$, where $f_i(k_i)$ is now expected output

- Apply Hughes and Majerovitz (2024), noting $\frac{dY}{dk} = r^{social} + \delta$

$$\log\left(Y^*/Y^{DE}\right) \approx \frac{1}{2} \cdot \mathcal{E} \cdot \log\left(1 + \frac{\text{Var}\left(r^{social}\right)}{\left(\mathbb{E}\left[r^{social}\right] + \delta\right)^2}\right)$$

- $\mathcal{E}$ is (negative) elasticity of output w.r.t. cost of capital $(r^{social} + \delta)$

Calibration of $\mathcal{E}$

▷ Back

- $\mathcal{E}_i$ is the elasticity of expected output with respect to the cost of capital
- Assume that $f(k, z) = z \cdot k^{\alpha}$ and there is no default, then

$$\mathcal{E} = \frac{\alpha}{1 - \alpha}$$

- $\alpha = \frac{1}{3}$ implies $\mathcal{E} = \frac{1}{2}$

Summary Statistics

▷ back

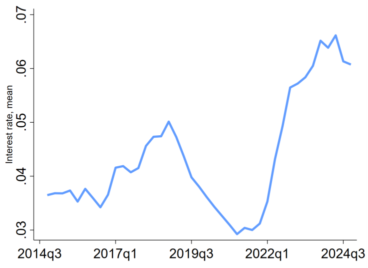
Table: Summary Statistics

	mean	sd	p10	p50	p90
Interest rate	4.17	1.69	2.21	3.93	6.59
Maturity (yrs)	6.85	4.64	3.00	5.00	10.25
ρ (%)	3.75	1.69	2.05	3.69	5.88
r^{firm} (%)	2.82	2.75	0.87	3.04	5.26
r^{social} (%)	3.54	1.88	1.77	3.53	5.71
Prob. Default (%)	1.42	2.37	0.19	0.82	2.85
LGD (%)	34.50	13.20	16.00	36.00	50.00
Loan amount (M)	10.77	68.81	1.11	2.55	22.64
Sales (M)	1,254.73	5,923.53	2.17	58.80	1,556.58
Assets (M)	1,770.83	8,956.78	1.06	35.52	1,792.00
Leverage (%)	72.03	24.57	42.57	71.17	100.00
Return on assets (%)	22.61	29.05	4.68	15.56	44.04
N Loans	62687				
N Firms	38587				
N Fixed Rate	31540				
N Variable Rate	31147				

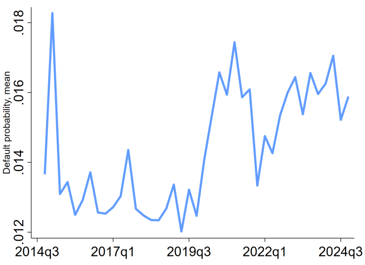
Time series for averages: Contractual Rate, Default, LGD

▷ back

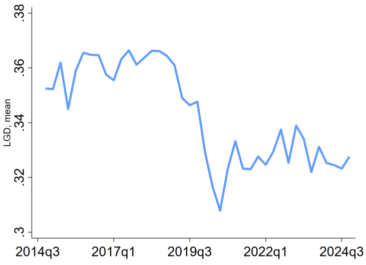
Contractual rate (fixed only)



Default Probability



LGD



- 2020-2021: Increase in default probability
- Modest decline in losses given default (better recovery)

Data Cleaning and Sample Construction

Sample period: We use FR Y-14Q Schedule H.1 data from 2014Q4 onward **Borrower Filters:**

- Drop loans without a Tax ID
- Keep only Commercial & Industrial loans to nonfinancial U.S. addresses
- Drop borrowers with NAICS codes:
  - 52 (Finance and Insurance), 92 (Public Administration)
  - 5312 (Real Estate Agents), 551111 (Bank Holding Companies)

Data Cleaning and Sample Construction

Loan Filters:

- Drop loans with:
  - Negative committed exposure
  - Utilized exposure exceeding committed exposure
  - Origination after or maturity before report date
- Keep only "vanilla" term loans (Facility type = 7)
- Drop loans with:
  - Mixed-rate structures
  - Maturity outside 110 years
  - Implausible interest rates or spreads (outside 1st99th percentile, or $> 50\%$)
  - Missing or invalid PD/LGD values (outside $[0, 1]$)
  - PD = 1 (flagged as in default)

Forward interest rate expectations

▷ Back

To estimate ρ for floating rate loans, we need estimates of $\mathbb{E}_0[r_t]$

- Floating rate loans charge reference rate + spread
- Approximate LIBOR/SOFR using Treasury forward yield curve estimates (Gürkaynak et al., 2007)
- Assume expectations hypothesis: long rates reflect expected short rates
- Back out $\mathbb{E}_0[r_t]$ for each loan, using treasury forward rate plus loan's spread

Lender's discount rate: fixed rate

▷ back

$$1 = \sum_{t=1}^{T} \left(\frac{P}{1+\rho}\right)^t \left[r + \frac{(1-P)}{P}(1-LGD)\right] + \left(\frac{P}{1+\rho}\right)^T$$

Let $x = \frac{P}{1+\rho}$ so

$$1 = \left(r + \frac{(1-P)}{P}(1-LGD)\right)\frac{x}{1-x}\left(1-x^T\right) + x^T$$

Guess that $1+\rho = (1+r)P + (1-P)(1-LGD)$

$$\frac{1-x}{x} = \frac{1}{x} - 1 = \frac{(1+r)P + (1-P)(1-LGD)}{P} - 1 = r + \frac{1-P}{P}(1-LGD)$$

And, therefore

$$1 = 1\left(1-x^T\right) + x^T$$

which validates the guess.

Firm cost of capital: model

▷ back

$$Q_t = \frac{\mathbb{E}_t \left[\mathcal{P}_{t+1} \left(\theta + (1-\theta)\, Q_{t+1} \right) + (1 - \mathcal{P}_{t+1})\, \phi k_{t+1}/b_{t+1} \right]}{1+\rho}$$

Note that

$$\begin{aligned} Q_t &= Q_t^P + Q_t^D \\ Q_t^P &= \frac{\mathbb{E}_t \left[\mathcal{P}_{t+1} \left(\theta + (1-\theta)\, Q_{t+1} \right) \right]}{1+\rho} \\ Q_t^D &= \frac{\mathbb{E}_t \left[(1 - \mathcal{P}_{t+1})\, \phi k_{t+1}/b_{t+1} \right]}{1+\rho} \end{aligned}$$

That is, we strip the bond into the payment in repay $\left(Q_t^P\right)$ and the payment in default $\left(Q_t^D\right)$. Then:

$$\Lambda = \frac{\mathbb{E}_t \left[(1 - \mathcal{P}_{t+1})\, \phi k_{t+1}/b_{t+1} \right]}{\mathbb{E}_t \left[\mathcal{P}_{t+1} \left(\theta + (1-\theta)\, Q_{t+1} \right) \right]} = \frac{Q_t^D}{Q_t^P}$$

Firm cost of capital: measurement

▷ back

The firm defaults with probability $(1-P)$ and the lender recovers $(1-LGD)$. Hence

$$Q_t^{D,data} = \frac{(1-P)(1-LGD)}{1+\rho}$$

For the payment portion notice that at issuance we have the following condition

$$1 = \sum_{s=1}^{T}\left[\frac{P^s\mathbb{E}_t\left[r_{t+s}\right] + P^{s-1}(1-P)(1-LGD)}{(1+\rho)^s}\right] + \frac{P^T}{(1+\rho)^T}$$

$$1 = \frac{(1-P)(1-LGD)}{1+\rho} + P\frac{\mathbb{E}_t\left[r_{t+1}\right]}{1+\rho} + \left(\sum_{s=2}^{T}\left[\frac{P^s\mathbb{E}_t\left[r_{t+s}\right] + P^{s-1}(1-P)(1-LGD)}{(1+\rho)^s}\right] + \frac{P^T}{(1+\rho)^T}\right)$$

So, we can define $Q_t^{P,data}$ as $1 = Q_t^{P,data} + Q_t^{D,data}$ so $Q_t^{P,data} = 1 - Q_t^{D,data}$. Finally

$$\Lambda^{data} = \frac{Q_t^{D,data}}{Q_t^{P,data}} = \frac{(1-P)(1-LGD)}{1+\rho-(1-P)(1-LGD)}$$

Decomposing misallocation

▷ Back

Counterfactual I: What if all lenders have the same $\bar{\rho}$?

$$1 + r^{cf,I}_{social} = \overline{(1+\rho)\mathcal{M}} + (lev - \mathcal{M}) \cdot PD \cdot (1 - LGD)$$

Heterogeneity in r^{cf}_{social} → Misallocation due to financial frictions

Counterfactual II: what if we equalize financial frictions?

$$1 + r^{cf,II}_{social} = (1+\rho)\,\mathcal{M} + \overline{(lev - \mathcal{M}) \cdot PD \cdot (1 - LGD)}$$

Heterogeneity in r^{cf}_{social} → Misallocation due to heterogeneous cost of capital

Variance decomposition

- Decompose total variance in: time, firm, bank, and error
- Keep firms with 5 or more securities

	Time	Bank	Firm	Loan
Contractual rate	71.88	1.63	13.45	13.04
Lender discount rate, ρ	61.94	3.08	14.02	20.96
Firm cost of capital, r^{firm}	33.23	4.25	20.12	42.4
Social cost of capital, r^{social}	53.84	3.87	16.21	26.08
N Firms	1681			
N Loans	14738			

Table: Variance decomposition of interest rates and cost of capital (ρ, r^{firm}, and r^{social})

Estimating $\mathcal{M}$

▷ back

$$\mathcal{M} = \frac{1 - \gamma \times \frac{Qb'}{k'} \times \frac{\partial \log Q}{\partial \log k'}}{1 + \gamma \times \times \frac{\partial \log Q}{\partial \log b'}}$$

Given estimates for the function Q, γ, and firm leverage Qb'/k' we can compute $\mathcal{M}$

1. Loans are modeled as perpetuities that decay at a geometric rate θ, we can write Q as the present value of all future payments, discounted at the contractual interest rate r:

$$Q = \frac{\theta + (1-\theta)Q}{1+r} = \frac{\theta}{r+\theta}$$

   r is directly observed in the data, and we can approximate $\theta = 1/T$

2. Guess a functional approximation $Q(z, k, b, \rho)$
3. Estimate $\log \hat{Q}(z, k, b, \rho)$ for every loan origination; compute partial derivatives
4. At steady state, $\gamma = \theta = 1/T$

Estimating $\mathcal{M}$: Q elasticities

▷ back

- We approximate (the log of) Q as a polynomial of investment, borrowing, productivity and ρ
- Investment: tangible assets
- Borrowing: total debt owed by the firm at loan origination
- Productivity: sales over tangible assets (Hsieh and Klenow, 2009)
- Approximation:

$$
\begin{aligned}
\log Q_i = {} & \alpha + \beta_k \log k_i + \beta_b \log b_i + \beta_z \log z_i + \beta_\rho \rho_i \\
& + \beta_{k,k} (\log k_i)^2 + \beta_{k,b} \log k_i \times \log b_i + \beta_{k,z} \log k_i \times \log z_i + \beta_{k,\rho} \log k_i \times \rho_i \\
& + \beta_{b,b} (\log b_i)^2 \beta_{b,z} \log b_i \times \log z_i + \beta_{b,\rho} \log b_i \times \rho_i \\
& + \beta_{z,z} (\log z_i)^2 \beta_{z,\rho} \log z_i \times \rho_i + \beta_{\rho,\rho} (\rho_i)^2 \epsilon_i
\end{aligned}
$$

- Compute the partial derivatives of $\log Q$ with respect to investment and borrowing.

Estimating $\mathcal{M}$: results

▷ back

- The distribution is extremely concentrated around 1.
- The mean is equal to 0.996 and the median to 0.997, with a standard deviation of 0.006.
- The two measures of misallocation are extremely similar
- Taken together, these results suggest that our assumption that $\mathcal{M} = 1$ is a good one.

Approximating misallocation

▷ back

- Recovery rates from the World Banks Doing Business report
- Approximate r^{social} with ρ in the SS for misallocation
- Use the fixed rate formula for ρ and assume that (P, LGD) are constant across firms
- Approximated cost of misallocation for the US is similar to the actual cost

References I

[ ]Abhijit V. Banerjee and Esther Duflo. Chapter 7 growth theory through the lens of development economics. In Handbook of Economic Growth, pages 473–552. Elsevier, 2005. doi: 10.1016/s1574-0684(05)01007-5.

[ ]Tiago V Cavalcanti, Joseph P Kaboski, Bruno S Martins, and Cezar Santos. Dispersion in financing costs and development. Technical report, National Bureau of Economic Research, 2024.

[ ]Joel M. David, Lukas Schmid, and David Zeke. Risk-adjusted capital allocation and misallocation. Journal of Financial Economics, 145(3):684–705, 2022. ISSN 0304-405X. doi: https://doi.org/10.1016/j.jfineco.2022.06.001. URL https://www.sciencedirect.com/science/article/pii/S0304405X22001398.

[ ]Miguel Faria-e-Castro, Samuel Jordan-Wood, and Julian Kozlowski. An Empirical Analysis of the Cost of Borrowing. Working Papers 2024-016, Federal Reserve Bank of St. Louis, July 2024. URL https://ideas.repec.org/p/fip/fedlwp/98542.html.

[]Simon Gilchrist, Jae W. Sim, and Egon Zakrajsek. Misallocation and financial market frictions: Some direct evidence from the dispersion in borrowing costs. Review of Economic Dynamics, 16(1):159–176, January 2013. ISSN 1094-2025. doi: 10.1016/j.red.2012.11.001.

[ ]Niels Joachim Gormsen and Kilian Huber. Corporate Discount Rates. June 2023. doi: 10.3386/w31329.

[ ]Niels Joachim Gormsen and Kilian Huber. Firms Perceived Cost of Capital. June 2024. doi: 10.3386/w32611.

References II

[]Refet S. Gürkaynak, Brian Sack, and Jonathan H. Wright. The u.s. treasury yield curve: 1961 to the present. Journal of Monetary Economics, 54(8):2291–2304, 2007. ISSN 0304-3932. doi: https://doi.org/10.1016/j.jmoneco.2007.06.029. URL https://www.sciencedirect.com/science/article/pii/S0304393207000840.

[ ]Chang-Tai Hsieh and Peter J. Klenow. Misallocation and manufacturing tfp in china and india. Quarterly Journal of Economics, 124(4):1403–1448, November 2009. doi: 10.1162/qjec.2009.124.4.1403.

[ ]David Hughes and Jeremy Majerovitz. Measuring misallocation with experiments. Technical report, 2025.

[ ]Diego Restuccia and Richard Rogerson. Policy distortions and aggregate productivity with heterogeneous establishments. Review of Economic Dynamics, 11(4):707–720, October 2008. doi: 10.1016/j.red.2008.05.002.