

Beauty Contests and the Term Structure

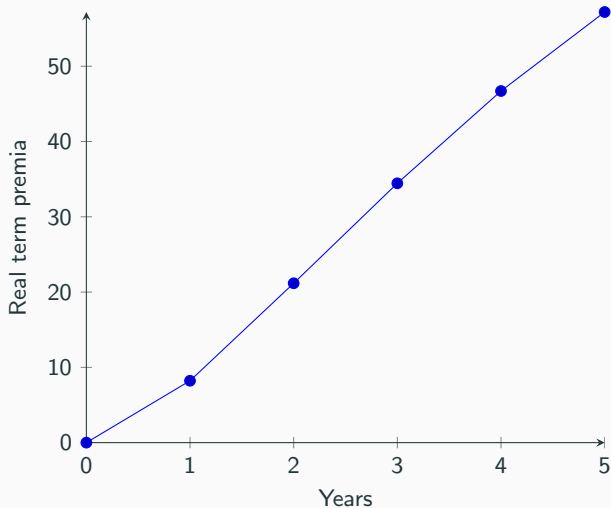
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Discussion by Julian Kozlowski, Federal Reserve Bank of St. Louis

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Real Term Premia



Source: term premium on Treasuries, Adrian et. al. (2013).

Real term premia: Inflation risk, liquidity, **information asymmetries**, etc.

This paper: Nice approach on information asymmetries and the term structure

Contributions:

1. Novel decomposition of the term structure
2. Beauty contest model → role of information on the term premia
3. Quantitative evaluation

1. Decomposition

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- **Real term premia** of a 2-periods bond

i_t : one-period rate, m_{t+1} : SDF

$$\phi_t^{(2)} \equiv \frac{1}{2} \left(\underbrace{E_t(e^{-i_t} e^{-i_{t+1}})}_{\text{risk-neutral expectation hypothesis}} - \underbrace{E_t(m_{t+1} m_{t+2})}_{\text{price for risk-averse household}} \right)$$

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- **Decomposition**

$$\phi^{(2)} = \frac{1}{2} \left(\underbrace{-\text{Cov} \left(m_{t+1}, m_{t+2} \right)}_{\text{covariances of successive realised SDFs}} + \underbrace{\text{Cov} \left(E_t \left(m_{t+1} \right), E_{t+1} \left(m_{t+2} \right) \right)}_{\text{covariances of successive expectations of SDFs}} \right)$$

- **Role for information on the term premia!**

- Log consumption follows

$$a_t = x_t + \eta_t \quad \eta_t \sim N(0, \sigma_\eta^2)$$

$$x_t = \rho x_{t-1} + \epsilon_t \quad \epsilon_t \sim N(0, \sigma_\epsilon^2)$$

- Risk averse household with CRRA coefficient σ

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- Covariances

$$\begin{aligned}-\text{Cov}(m_{t+1}, m_{t+2}) &= (\beta\sigma)^2 \left(\frac{1-\rho}{1+\rho} \sigma_\epsilon^2 + \sigma_\eta^2 \right) \\ \text{Cov}(E_t(m_{t+1}), E_{t+1}(m_{t+2})) &= (\beta\sigma)^2 \frac{1-\rho}{1+\rho} \rho \sigma_\epsilon^2\end{aligned}$$

- Idiosyncratic shock η : only affects realized SDFs
- Persistent shock ϵ : lower effect on expected SDFs
- Role of σ

$$\phi_{FI}^{(2)} = \frac{1}{2} (\beta\sigma)^2 \left[(1 - \rho)\sigma_\epsilon^2 + \sigma_\eta^2 \right]$$

- Term premia: $\beta(+)$, $\sigma(+)$, $\rho(-)$, $\sigma_\epsilon(+)$, and $\sigma_\eta(+)$
- Risk aversion: term premia increases at rate σ^2
- Zero term premia if
 - risk-neutral ($\sigma = 0$)
 - SDF is iid ($\sigma_\eta^2 = 0$ and $\rho = 1$)
- The term premia is always positive
 - Is this true only in the simple model, or is it a general property?
 - Is this a desirable property?
 - On average it is positive... but cannot account for inverted yield curves

2. Beauty contest

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- Signal $s_{i,t} = a_t + n_t + n_{i,t}$ with common and idiosyncratic noise AR(1)
- Expected SDF $\hat{m}_{i,t} = \beta(1 + (1 - \rho)\hat{x}_{i,t})$
- Beauty contest

$$\hat{x}_{i,t} = \arg \min (1 - \omega) E(\hat{x}_{i,t} - x_t)^2 - \omega E \left(\int_0^1 \hat{x}_{j,t} dj \right) \hat{x}_{i,t}$$

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- Amplification: Excess term premia

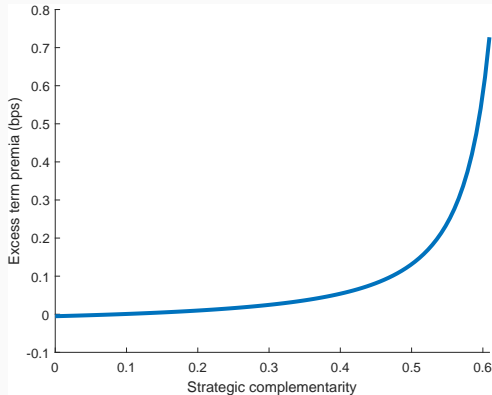
$$E\psi_{BC}^{(2)} - E\psi_{FI}^{(2)} = \frac{\beta^2 \rho (1 - \rho)}{2} \frac{1 - \rho}{1 + \rho} \left[\theta^2 (\sigma_\xi^2 + \sigma_\zeta^2) - (1 - \theta^2) \sigma_\epsilon^2 \right]$$

$$\theta = \frac{(1 - \omega) \sigma_\epsilon^2}{(1 - \omega) (\sigma_\epsilon^2 + \sigma_\xi^2 + \sigma_\zeta^2) - \frac{\omega}{2} (\sigma_\epsilon^2 + \sigma_\xi^2)}$$

ϵ is the shock to x_t and ξ and ζ are the shocks to the common and idiosyncratic noises

→ Positive excess term premia when noise is sufficiently large

Excess term premium



- Needs a large strategic complementarity to generate sizable amplification
- Note the convexity in $\omega \rightarrow$ important for quantitative results
- Why $\omega = 0$ does not nest the full information allocation?

- **Beauty contest:** *“The strategic complementarity in our model could be rationalized by fears that the household might suffer a **liquidity shock** and so need to liquidate their bond holdings within the period, in which case they would be interested in the expected price on liquidation”*

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- **Search-theoretic models** of the term premium
 - Geromichalos Herrenbrueck and Salyer 2016, Kozlowski 2018
 - Market structure of bonds → Trading over-the-counter
 - Difference in valuations → motives for trade (information asymmetries?)
 - Generate sizable term premia

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 - Generate sizable term premia
- **Same underlying forces?**
 - Important for measurement
 - Is this a paper about the **liquidity component of term premia?**
 - Might help the quantitative application

3. Quantitative evaluation

3. General model and quantitative analysis

Strategy

- Target one point (4-quarters) and test at longer maturities
- Theoretical results are about the level (2-period model) not the slope
What are we testing in the data?
- Potential solutions:
 - Extend the results for 3-periods to study the amplification on the slope
 - Or use alternative strategy looking at cross-sectional variation

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Data

- Target and test for all the term premia
- But there are other components: inflation risk, liquidity, etc
- What components of the data is the model trying to explain?
- Maybe should focus on the liquidity component only
- For example as measured in Krishnamurthy Vissing-Jorgensen 2012

- Strategic complementarity
 - Estimation sets $\omega = 0.66$ at the upper bound that the model can support
 - Maximum amplification here (remember convexity)
 - Is ω identified by the moments from the SPF or by the term premia?
 - Some quantitative identification exercises would be helpful here
e.g. how the target moments change with ω

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e.g. how the target moments change with ω
- Risk aversion $\sigma = 4$ to match the level of the term premium at 4 quarters
- Slope:

$$tp_n = \alpha + \beta n + \epsilon_n$$

	Data	Beauty contest	Full Info	$\omega = 0$
Slope β (year)	11.85	4.69	1.37	1.20
% explained		40%	12%	10%

- Beauty contest generates an amplification of about 5bps per year
- Is this too much or too little for the liquidity component?
It is in the ballpark of other estimates (e.g., Kozlowski 2018)

Cross-sectional variation and testable implications:

- Derive testable implications of the theory across different assets
- Assets with more information asymmetries should have larger term premia
- What do we know about this implication in the cross-section of assets?
- Might be able to exploit time-series variation as well
- Yield curve inversion? What is the role of information asymmetries?