Beauty Contests and the Term Structure

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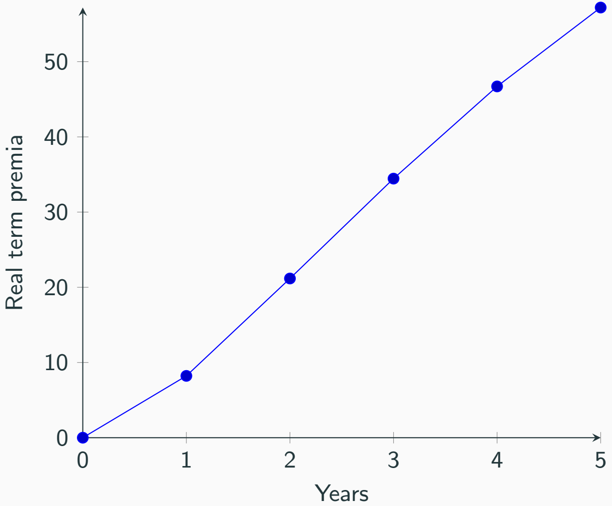
Discussion by Julian Kozlowski, Federal Reserve Bank of St. Louis

Expectations in Dynamic Macroeconomics Model, Birmingham, August 2018

Real Term Premia



Source: term premium on Treasuries, Adrian et. al. (2013).

Real term premia: Inflation risk, liquidity, **information asymmetries**, etc.

Beauty Contests and the Term Structure

This paper: Nice approach on information asymmetries and the term structure

Contributions:

1. Novel decomposition of the term structure
2. Beauty contest model $\rightarrow$ role of information on the term premia
3. Quantitative evaluation

1. Decomposition

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- Real term premia of a 2-periods bond
  i_t: one-period rate, m_{t+1}: SDF

$$\phi_t^{(2)} \equiv \frac{1}{2}\left(\underbrace{E_t\left(e^{-i_t}e^{-i_{t+1}}\right)}_{\text{risk-neutral expectation hypothesis}} - \underbrace{E_t\left(m_{t+1}m_{t+2}\right)}_{\text{price for risk-averse household}} \right)$$

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- Decomposition

$$\phi^{(2)} = \frac{1}{2}\left(\underbrace{-Cov\left(m_{t+1}, m_{t+2}\right)}_{\text{covariances of successive realised SDFs}} + \underbrace{Cov\left(E_t\left(m_{t+1}\right), E_{t+1}\left(m_{t+2}\right)\right)}_{\text{covariances of successive expectations of SDFs}} \right)$$

- Role for information on the term premia!

Simple model

- Log consumption follows

$$a_t = x_t + \eta_t \qquad \eta_t \sim N(0, \sigma_\eta^2)$$
$$x_t = \rho x_{t-1} + \epsilon_t \qquad \epsilon_t \sim N(0, \sigma_\epsilon^2)$$

- Risk averse household with CRRA coefficient σ

$$m_{t+1} = \beta\left(1 + \sigma\left(a_t - a_{t+1}\right)\right)$$

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- Covariances

$$-Cov\left(m_{t+1}, m_{t+2}\right) = \left(\beta\sigma\right)^2 \left(\frac{1-\rho}{1+\rho}\sigma_\epsilon^2 + \sigma_\eta^2\right)$$
$$Cov\left(E_t\left(m_{t+1}\right), E_{t+1}\left(m_{t+2}\right)\right) = \left(\beta\sigma\right)^2 \frac{1-\rho}{1+\rho}\rho\sigma_\epsilon^2$$

  - Idiosyncratic shock η: only affects realized SDFs
  - Persistent shock ϵ: lower effect on expected SDFs
  - Role of σ

Full information term premia

$$\phi_{FI}^{(2)} = \frac{1}{2} (\beta\sigma)^2 \left[(1-\rho)\sigma_\epsilon^2 + \sigma_\eta^2\right]$$

- Term premia: $\beta(+)$, $\sigma(+)$, $\rho(-)$, $\sigma_\epsilon(+)$, and $\sigma_\eta(+)$
- Risk aversion: term premia increases at rate σ^2
- Zero term premia if
  - risk-neutral ($\sigma = 0$)
  - SDF is iid ($\sigma_\eta^2 = 0$ and $\rho = 1$)
- The term premia is always positive
  - Is this true only in the simple model, or is it a general property?
  - Is this a desirable property?
  - On average it is positive... but cannot account for inverted yield curves

2. Beauty contest

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- Signal $s_{i,t} = a_t + n_t + n_{i,t}$ with common and idiosyncratic noise AR(1)
- Expected SDF $\hat{m}_{i,t} = \beta\left(1 + (1-\rho)\hat{x}_{i,t}\right)$
- Beauty contest

$$\hat{x}_{i,t} = \arg\min\,(1-\omega)\,E(\hat{x}_{i,t} - x_t)^2 - \omega E\left(\int_0^1 \hat{x}_{j,t}dj\right)\hat{x}_{i,t}$$

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- Amplification: Excess term premia

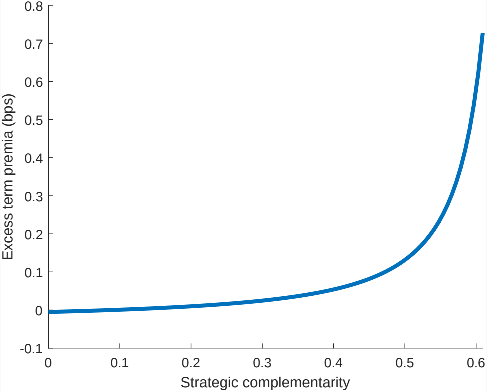
$$E\psi^{(2)}_{BC} - E\psi^{(2)}_{FI} = \frac{\beta^2}{2}\frac{\rho\,(1-\rho)}{1+\rho}\left[\theta^2\left(\sigma^2_\xi + \sigma^2_\zeta\right) - \left(1-\theta^2\right)\sigma^2_\varepsilon\right]$$

$$\theta = \frac{(1-\omega)\,\sigma^2_\varepsilon}{(1-\omega)\left(\sigma^2_\varepsilon + \sigma^2_\xi + \sigma^2_\zeta\right) - \frac{\omega}{2}\left(\sigma^2_\varepsilon + \sigma^2_\xi\right)}$$

ϵ is the shock to x_t and ξ and ζ are the shocks to the common and idiosyncratic noises

$\rightarrow$ Positive excess term premia when noise is sufficiently large

Excess term premium



- Needs a large strategic complementarity to generate sizable amplification
- Note the convexity in $\omega \rightarrow$ important for quantitative results
- Why $\omega = 0$ does not nest the full information allocation?

Microfoundations for Beauty Contest

- Beauty contest: *"The strategic complementarity in our model could be rationalized by fears that the household might suffer a* **liquidity shock** *and so need to liquidate their bond holdings within the period, in which case they would be interested in the expected price on liquidation"*

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- Search-theoretic models of the term premium
  - Geromichalos Herrenbrueck and Salyer 2016, Kozlowski 2018
  - Market structure of bonds → Trading over-the-counter
  - Difference in valuations → motives for trade (information asymmetries?)
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  - Generate sizable term premia
- Same underlying forces?
  - Important for measurement
  - Is this a paper about the **liquidity component of term premia**?
  - Might help the quantitative application

3. Quantitative evaluation

Strategy

- Target one point (4-quarters) and test at longer maturities
- Theoretical results are about the level (2-period model) not the slope
  What are we testing in the data?
- Potential solutions:
  - Extend the results for 3-periods to study the amplification on the slope
  - Or use alternative strategy looking at cross-sectional variation

3. General model and quantitative analysis

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Data

- Target and test for all the term premia
- But there are other components: inflation risk, liquidity, etc
- What components of the data is the model trying to explain?
- Maybe should focus on the liquidity component only
- For example as measured in Krishnamurthy Vissing-Jorgensen 2012

Quantitative results

- Strategic complementarity
  - Estimation sets $\omega = 0.66$ at the upper bound that the model can support
  - Maximum amplification here (remember convexity)
  - Is ω identified by the moments from the SPF or by the term premia?
  - Some quantitative identification exercises would be helpful here
    e.g. how the target moments change with ω

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- Risk aversion $\sigma = 4$ to match the level of the term premium at 4 quarters
- Slope:

$$tp_n = \alpha + \beta n + \epsilon_n$$

	Data	Beauty contest	Full Info	$\omega = 0$
Slope β (year)	11.85	4.69	1.37	1.20
% explained		40%	12%	10%

- Beauty contest generates an amplification of about 5bps per year
- Is this too much or too little for the liquidity component?
  It is in the ballpark of other estimates (e.g., Kozlowski 2018)

Alternative quantitative exploration

Cross-sectional variation and testable implications:

- Derive testable implications of the theory across different assets
- Assets with more information assymetries should have larger term premia
- What do we know about this implication in the corss-section of assets?
- Might be able to exploit time-series variation as well
- Yield curve inversion? What is the role of information asymmetries?