The Cost of Capital and Misallocation in the United States

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Cost of capital and misallocation

Research question: How does dispersion in the cost of capital affect misallocation?

Traditional misallocation approach:

- Strong assumptions about production functions (homogeneous Cobb-Douglas)
- Measure heterogeneity in marginal products from cross-sectional input data
- Estimate capital misallocation

Our macrofinance approach:

- Main idea: cost of capital equals marginal product of capital
- Combine credit registry data with model to carefully measure cost of capital
- Use dispersion in cost of capital to quantify welfare losses from imperfect credit markets

The cost of capital and misallocation in the US

▷ literature

Methodological contribution:

- Adapt a standard dynamic corporate finance model for measurement with loan-level data
- Derive a sufficient statistic for misallocation based on credit registry data

Empirical results for the US:

- Average cost of capital tracks treasury rates, with a spread
- Measures of cost of capital correlate with traditional measures of $ARPK_i$ at the firm level
- Credit markets seem quite efficient in normal times—misallocation losses of 0.9% of GDP
- Losses from misallocation increased to 1.8% of GDP in 2020-2021
- Possibly tied to mispricing of credit due to credit market interventions

Outline

Model

Welfare and Misallocation

Measurement with credit registry data

Misallocation in the US

Extensions & Robustness

Model

Borrower-Lender model

- Discrete time, infinite horizon
- Continuum of firms, each matched with a lender
- No aggregate risk (for now - work in progress!)

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- Produce output $f(k_i, z_i)$
- Invest in capital k_i
- Long-term debt b_i
- Limited liability

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- Discount rate ρ_i
- Recover $\phi_i k_i$ in default
- Lenders price loans to break even

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Key question: how do heterogeneity in ρ_i and financial frictions distort the allocation of capital?

Model

Firm value function:

$$V_i\left(k_i, b_i, z_i\right) = \max_{k_i', b_i'} \pi_i\left(k_i, b_i, z_i, k_i', b_i'\right) + \beta \mathbb{E}\overbrace{\left[\max\left\{V_i\left(k_i', b_i', z_i'\right), 0\right\} \middle| z_i\right]}^{\text{Limited liability}}$$

Firm profits:

$$\pi_i\left(k_i, b_i, z_i, k_i', b_i'\right) = f\left(k_i, z_i\right) + \left(1-\delta\right) k_i - k_i' - \theta b_i + Q_i\left(k_i', b_i', z_i\right)\left[b_i' - \left(1-\theta_i\right) b_i\right]$$

Price of debt:

$$Q_i\left(k_i', b_i', z_i\right) = \frac{\mathbb{E}\left\{\overbrace{\mathcal{P}_i\left(k_i', b_i', z_i'\right)}^{\text{repayment prob.}}\left[\theta_i + \left(1-\theta_i\right) Q_i\left(k_i'', b_i'', z_i'\right)\right] + \left(1-\mathcal{P}_i\left(k_i', b_i', z_i'\right)\right)\overbrace{\frac{\phi_i k_i'}{b_i'}}^{\text{recovery}} \middle| z_i\right\}}{\underbrace{1+\rho_i}_{\text{lender discount rate}}}$$

Firm optimality: cost of capital vs. expected MRPK

▷ details

- **Cost of capital:**

$$\underbrace{\frac{\mathbb{E}\left[\mathcal{P}_i'(\theta_i + (1-\theta_i)Q_i') \mid z_i\right]}{Q_i}}_{1+r_i^{\text{firm}}} \times \underbrace{\left[\frac{1 - \frac{\partial Q_i}{\partial k_i'}[b_i' - (1-\theta_i)b_i]}{1 + \frac{\partial Q_i}{\partial b_i'}\frac{[b_i' - (1-\theta_i)b_i]}{Q_i}}\right]}_{\mathcal{M}_i}$$

- $1 + r_i^{\text{firm}}$: implied interest rate perceived by the firm.
- $\mathcal{M}_i$: price-impact term capturing how (k_i', b_i') affects the debt price Q_i.

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- $1 + r_i^{\text{firm}}$: implied interest rate perceived by the firm.

- $\mathcal{M}_i$: price-impact term capturing how (k_i', b_i') affects the debt price Q_i.

- **Optimality:** Optimal investment equates the firms financing cost to the expected MRPK

$$(1 + r_i^{\text{firm}})\,\mathcal{M}_i = \underbrace{\mathbb{E}\left[\mathcal{P}_i'\left(f_k(k_i', z_i') + 1 - \delta\right) \,\middle|\, z_i\right]}_{\text{expected MRPK}}$$

- Measurement idea: dispersion in r_i^{firm} (from loan data) $\Rightarrow$ dispersion in MRPK $\Rightarrow$ misallocation.

Firm's cost of capital

Lemma 1 (Firm's cost of capital)

The firm's cost of capital is:

$$1 + r_i^{firm} = \frac{1 + \rho_i}{1 + \Lambda_i} \qquad \Lambda_i := \frac{\mathbb{E}\left[(1 - \mathcal{P}_i')\, \phi_i k_i' / b_i' \,\middle|\, k_i', b_i', z_i\right]}{\mathbb{E}\left[\mathcal{P}_i' \left(\theta + (1 - \theta_i) Q_i'\right) \,\middle|\, k_i', b_i', z_i\right]}$$

▷ *Proof*

- Λ_i is a wedge due to financial frictions, positive if lender recovers in default.
- In general, $r_i^{firm} < \rho_i$, since lender recovers some in default, but firm pays zero.

Welfare and Misallocation

Aggregate economy and welfare
Decentralized Equilibrium

$$Y^{DE} + (1-\delta)K^{DE} = \int_0^1 \mathbb{E}_t \left[\mathcal{P}^{DE}_{i,t+1}\left(f(k^{DE}_{i,t+1}, z_{i,t+1}) + (1-\delta)k^{DE}_{i,t+1}\right) + (1-\mathcal{P}^{DE}_{i,t+1}) \cdot \phi k^{DE}_{i,t+1}\right] \mathrm{d}i$$

Aggregate economy and welfare
Decentralized Equilibrium

$$Y^{DE} + (1-\delta)K^{DE} = \int_0^1 \mathbb{E}_t \left[\mathcal{P}^{DE}_{i,t+1}\left(f(k^{DE}_{i,t+1}, z_{i,t+1}) + (1-\delta)k^{DE}_{i,t+1}\right) + (1-\mathcal{P}^{DE}_{i,t+1}) \cdot \phi k^{DE}_{i,t+1}\right] \mathrm{d}i$$

Planner's problem: Intensive-margin misallocation

- Redistribute capital taking exit decisions and aggregate capital as given.
- Misallocation on the intensive margin is the main focus of the misallocation literature (e.g. Hsieh and Klenow, 2009; Restuccia and Rogerson, 2008).

$$\max_{\left\{k^*_{i,t+1}\right\}_i} \int_0^1 \mathbb{E}_t \left[\mathcal{P}^{DE}_{i,t+1}\left(f(k^*_{i,t+1}, z_{i,t+1}) + (1-\delta)k^*_{i,t+1}\right) + (1-\mathcal{P}^{DE}_{i,t+1}) \cdot \phi k^*_{i,t+1}\right] \mathrm{d}i$$

$$\text{s.t.} \quad \int_0^1 k^*_{i,t+1} \mathrm{d}i = K^{DE}_{t+1}$$

▷ full planner problem

Social return on capital

- Define the social marginal product of capital at firm i, $r^{social}_{i,t}(k)$

$$1 + r^{social}_{i,t}(k) \equiv \mathbb{E}\left[\mathcal{P}^{DE}_{i,t+1}\left(f_k\left(k, z_{i,t+1}\right) + 1 - \delta\right) + \left(1 - \mathcal{P}^{DE}_{i,t+1}\right)\phi_i\right]$$

- Social return includes expected recovery in default, which is not taken into account by the firm.

- Planner Optimality: The planner chooses $k^*_{i,t+1}$ to equalize $r^{social}_{i,t}(k^*_{i,t+1})$ across firms.

- Equilibrium: Dispersion in $r^{social}_{i,t}(k^{DE}_{i,t+1}) \rightarrow$ misallocation.

Misallocation in the decentralized equilibrium

- In the decentralized equilibrium:

$$(1 + r_{i,t}^{firm})\mathcal{M}_{i,t} = \mathbb{E}_t[\mathcal{P}_{i,t+1}^{DE}(f_k(k_{i,t+1}^{DE}, z_{i,t+1}) + 1 - \delta)]$$

- Hence:

$$1 + r_{i,t}^{social}(k_{i,t+1}^{DE}) = (1 + r_{i,t}^{firm})\mathcal{M}_{i,t} + \left(1 - \mathcal{P}_{i,t+1}^{DE}\right)\phi_i$$

- Measurement idea: dispersion in $r_{i,t}^{\text{firm}}$ (from loan data) $\Rightarrow$ dispersion in $r_{i,t}^{social}$ $\Rightarrow$ misallocation.

Sufficient statistic for misallocation

Proposition 1 (Misallocation)

Misallocation can be measured with $\mathbb{E}\left[r_i^{social}\right]$ *and* $Var\left(r_i^{social}\right)$ *as*

$$\log\left(Y^*/Y^{DE}\right) \approx \frac{1}{2} \cdot \mathcal{E} \cdot \log\left(1 + \frac{Var\left(r_i^{social}\right)}{\left(\mathbb{E}\left[r_i^{social}\right] + \delta\right)^2}\right)$$

▷ *Proof*

- Extends Hughes and Majerovitz (2025) to a dynamic economy with default
- Measures intensive-margin misallocation.
- Calibrate $\mathcal{E} = \frac{1}{2}$ (elasticity of output w.r.t. $r^{social} + \delta$) and $\delta = 0.06$ ▷ Calibration
- **Next:** show how to measure r_i^{social} using credit registry data

Measurement with credit registry data

Measurement with credit registry data

▷ Summary stats ▷ cleaning details

- Data: FR Y-14Q (Schedule H.1)
- Universe: all C&I loans $\geq$ \$1M (2014Q4 - 2024Q4)
- Coverage: top 40 BHCs ($\approx$ 91 % of C&I lending)
- Variables: interest rate, spread, PD, LGD, maturity, type.
- Focus on term loans issued to non-government, non-financial US firms

Pricing term loans

For a loan i originated at t, the break-even condition for a lender with discount rate $\rho_{i,t}$ is

$$1 = \sum_{s=1}^{T_{i,t}} \left[\frac{(P_{i,t})^s \cdot \mathbb{E}_t\left(r_{i,t,s}\right) + (P_{i,t})^{s-1} \cdot (1 - P_{i,t}) \cdot (1 - LGD_{i,t})}{(1+\rho_{i,t})^s \cdot \mathbb{E}_t(\Pi_{t,s})} \right] + \frac{(P_{i,t})^{T_{i,t}}}{(1+\rho_{i,t})^{T_{i,t}} \cdot \mathbb{E}_t(\Pi_{t,T_{i,t}})}$$

- $T_{i,t}$: maturity
- $\mathbb{E}_t[r_{i,t,s}]$: fixed rate or spread over benchmark rate (Gürkaynak et al., 2007) ▷ forward rates
- $P_{i,t}$: repayment probability (constant over time)
- $LGD_{i,t}$: loss given default (constant over time)
- $\mathbb{E}_t(\Pi_{t,s})$: total expected inflation from t to s (Cleveland Fed)
- $\Rightarrow$ Solve for lender's discount rate: $\rho_{i,t}$ ▷ fixed real rate

Measuring firm and social cost of capital

Lemma 2 (Firm and social cost of capital)

We can write the firm cost of capital as

$$1 + r_{i,t}^{firm} = (1 + \rho_{i,t}) - (1 - P_{i,t})(1 - LGD_{i,t})$$

The social cost of capital can be written as:

$$\begin{aligned} 1 + r_{i,t}^{social} &= (1 + r_{i,t}^{firm})\mathcal{M}_{i,t} + (1 - P_{i,t})(1 - LGD_{i,t})lev_{i,t} \\ &= \underbrace{(1 + \rho_{i,t})\,\mathcal{M}_{i,t}}_{\text{lender discount rate}} + \underbrace{(lev_{i,t} - \mathcal{M}_{i,t}) \cdot (1 - P_{i,t}) \cdot (1 - LGD_{i,t})}_{\text{wedge due to financial frictions}} \end{aligned}$$

▷ *Proof*

$r^{firm} \leq r^{social} \leq \rho$: firms face lower perceived cost of capital because lenders recover in default

▷ financial frictions

Sufficient statistic for misallocation

$$\log\left(Y_t^*/Y_t^{DE}\right) \approx \frac{1}{2}\cdot\mathcal{E}\cdot\log\left(1+\frac{\text{Var}\left(r_{i,t}^{social}\right)}{\left(\mathbb{E}\left[r_{i,t}^{social}\right]+\delta\right)^2}\right)$$

$$1+r_{i,t}^{social} = (1+\rho_{i,t})\,\mathcal{M}_{i,t} + (lev_{i,t}-\mathcal{M}_{i,t})\cdot(1-P_{i,t})\cdot(1-LGD_{i,t})$$

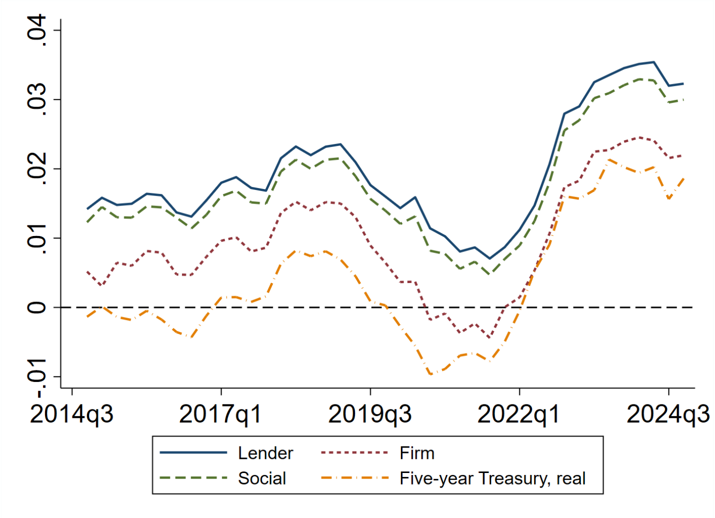
- Set $\mathcal{M}_{i,t}=1$; reasonable approximation given our model

▷ Estimate $\mathcal{M}$

- Can measure misallocation directly with credit registry data!
- Dispersion in $r_{i,t}^{social}$ comes from:
  1. Dispersion in lender's discount rate, $\rho_{i,t}$
  2. Dispersion in financial frictions wedge
  3. Covariance between $\rho_{i,t}$ and financial frictions wedge

Misallocation in the US

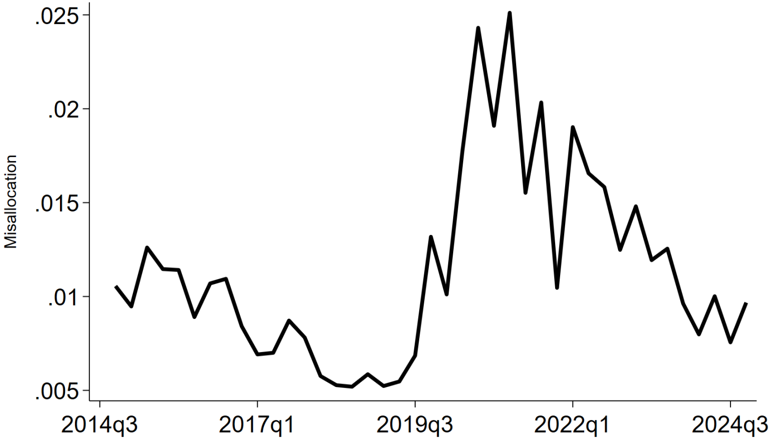
Time series for average discount rate, firm and social cost of capital



- Rates follow 5y UST
- Financial frictions:
  $\mathbb{E}\left[r_i^{social}\right] > \mathbb{E}\left[r_i^{firm}\right]$
- $\mathbb{E}\left[r_i^{social}\right] \approx \mathbb{E}\left[\rho_i\right]$

▷ Rate estimates

Output losses from capital misallocation



- About 0.9% before 2020
- $\uparrow$ to 1.8% in 2020-2021
- $\downarrow$ to 1.2% in 2022-2024

The 2020–2021 increase in misallocation

1. Driven by dispersion in lender discount rates ρ_i, not financial frictions.

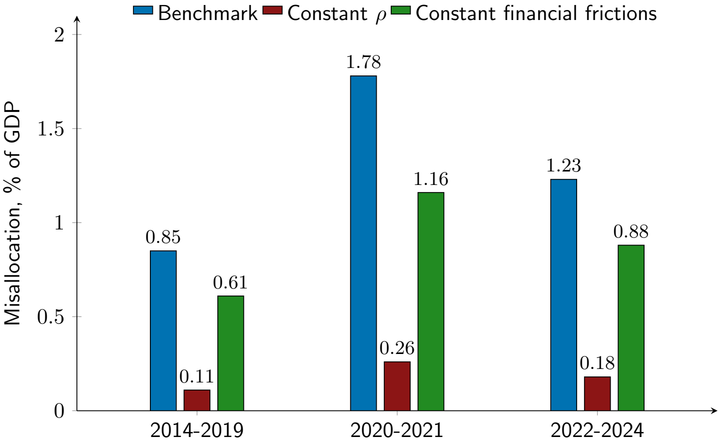
2. Sharp rise in the coefficient of variation of ρ_i.

3. Variance of ρ_i increases due to increased dispersion of expected losses.

$\Rightarrow$ Likely linked to policy-induced underpricing of risky loans and implicit guarantees.

1. The 2020-21 rise in misallocation was driven by $\{\rho_i\}$

▷ details



- Main driver: dispersion in lender discount rates
- Interaction between ρ_i and financial frictions $(0.85 > 0.11 + 0.61)$

2. The CV of ρ_i increased during 2020-21

- As policy rates decreased in 2020-21, so did mean ρ_i
- Standard deviation of ρ_i increased during this period

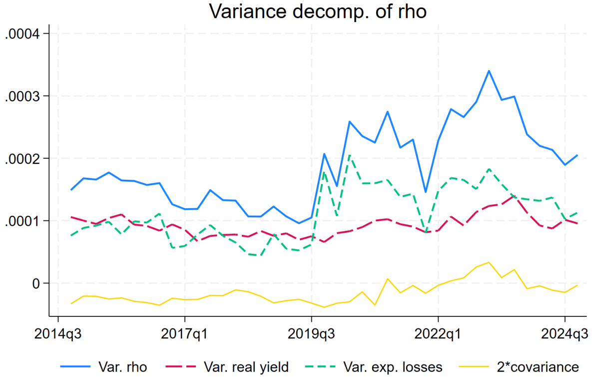
$\Rightarrow$ 2. Sharp rise in the coefficient of variation of ρ_i

3. Variance of ρ related to variance of expected losses

- Compute real yield $\rho^*_{i,t}$: the rate implied if no default

▷ real yield

- Decomposition: $\rho_i = \underbrace{\rho^*_i}_{\text{real yield}} + \underbrace{[\rho_i - \rho^*_i]}_{\text{exp. losses}}$



Variance of ρ_i:

$\mathbb{V}\,[\text{yield}] + \mathbb{V}\,[\text{exp. losses}] + 2\mathbb{C}\,[\text{yield, exp. losses}]$

- Increase in variance explained by exp. losses
- Likely linked to policy-induced underpricing of risky loans and implicit guarantees.

Extensions & Robustness

Extensions & Robustness

1. Estimate heterogeneous price-impact term $\mathcal{M}$. ▷ heterogeneous $\mathcal{M}$
2. Variance decomposition: dispersion accounted by bank, firm, loan. ▷ variance decomposition
3. Validate r^{social} using firm-level ARPK measures. ▷ details
4. Application to cross-country data. ▷ details

Work in progress ⚙

1. Aggregate risk
2. Quantitative model

Conclusion

- Framework to measure misallocation from credit registry data.

  1. Standard dynamic corporate finance model as measurement device
  2. Sufficient statistic for capital misallocation
  3. Uses standard credit registry variables $(r, P, LGD, T, \dots)$

- Application to U.S. credit registry data

  1. Estimate lender discount rates, firm-level cost of capital and social cost of capital
  2. Misallocation around 1% in normal times
  3. Rise in 2020-21, driven by increase in variance of expected losses

Credit markets in the US appear efficient, but crisis interventions can amplify misallocation.

Appendices

Related literature

▷ back

- **Measuring misallocation:**
  - Seminal work: Restuccia and Rogerson (2008), Hsieh and Klenow (2009)
  - Challenge: Standard methods rely on strong assumptions (Haltiwanger et al., 2018).
  - Recent advances: Experimental/quasi-experimental methods to recover marginal products directly (Carrillo et al., 2023; Hughes and Majerovitz, 2025).
  - Contribution: use heterogeneity in funding costs to measure dispersion in MRPK
- **Heterogeneity in the cost of capital:**
  - Developing countries: Banerjee and Duflo (2005), Cavalcanti, Kaboski, Martins, and Santos (2024)
  - US: Gilchrist, Sim, and Zakrajsek (2013), Gormsen and Huber (2023, 2024), Faria-e-Castro, Jordan-Wood, and Kozlowski (2024)
  - Contribution:
    - Estimate firm cost of capital using credit registry data, correcting for loan characteristics, etc.
    - Derive and estimate sufficient statistic for misallocation

Firm FOC: details

▷ back

Firm FOCs:

$$[k_i'] : \quad -1 + \frac{\partial Q_i(k_i', b_i', z_i)}{\partial k_i'}[b_i' - (1-\theta_i)b_i] + \beta \mathbb{E}\left\{ \mathcal{P}_i(k_i', b_i', z_i')[f_k(k_i', z_i') + 1 - \delta] \middle| z_i \right\} = 0$$

$$[b_i'] : \quad \frac{\partial Q_i(k_i', b_i', z_i)}{\partial b_i'}[b_i' - (1-\theta_i)b_i] + Q_i(k_i', b_i', z_i) - \beta \mathbb{E}\left\{ \mathcal{P}_i(k_i', b_i', z_i')[\theta_i + (1-\theta_i)Q_i(k_i'', b_i'', z_i')] \middle| z_i \right\}$$

$$= 0$$

Firm's cost of capital: proof

▷ back

$$\frac{1}{Q_t}\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]=\frac{(1+\rho)\,\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]+\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k'/b'\right]}$$

$$=(1+\rho)\left(1+\frac{\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k'/b'\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]}\right)^{-1}$$

$$=(1+\rho)\left(1+\Lambda\right)^{-1}$$

where

$$\Lambda\equiv\frac{\mathbb{E}_t\left[\left(1-\mathcal{P}_{t+1}\right)\phi k'/b'\right]}{\mathbb{E}_t\left[\mathcal{P}_{t+1}\left(\theta+(1-\theta)\,Q_{t+1}\right)\right]}$$

Aggregate economy and welfare

▷ back

- Aggregate resource constraint:

$$Y_{t+1} + (1-\delta)K_{t+1} = \int_0^1 \mathbb{E}_t \left[\mathcal{P}_{i,t+1} \left(f\left(k_{i,t+1}, z_{i,t+1}\right) + (1-\delta)k_{i,t+1}\right) + \left(1-\mathcal{P}_{i,t+1}\right) \cdot \phi_i k_{i,t+1}\right] \mathrm{d}i$$

- Let $\omega_{i,t}(S^t) \in \{0,1\}$ denote whether a firm operates or not
- Assume that existing firms are replaced by identical ones
- Planner's problem:

$$U^* = \max_{\left\{\{k_{i,t}(S^{t-1}),\omega_{i,t}(S^t)\}_{i\in[0,1]}\right\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \cdot u\left(C_t\right)$$

$$\text{s.t.} \qquad K_t = \int_0^1 k_{i,t}(S^{t-1})\mathrm{d}i$$

$$C_t + K_{t+1} = Y_t + (1-\delta)K_t$$

$$\omega_{i,t+1}\left(S^{t+1}\right) \leq \omega_{i,t}\left(S^t\right) \; \forall S^t \subset S^{t+1}, \forall i$$

Aggregate economy and welfare, cont'd

▷ back

- Can separate planner's problem into outer (dynamic) and inner (static) problems:

$$U^* = \max_{\left\{K_t, \{\omega_{i,t}(S^t)\}_{i\in[0,1]}\right\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t \cdot u\left(\left(\max_{\left\{\{k_{i,t}(S^{t-1})\}_{i\in[0,1]}\right\}_{t=1}^{\infty}} Y_t\right) - I_t\right)$$

- Rewrite inner problem as:

$$Y_t^*\left(K_t, \{\omega_{it}\}_{i\in[0,1]}\right) = \max_{\left\{k_{i,t}^*\right\}_{i\in[0,1]}} \int_0^1 \mathbb{E}_{t-1}\left\{\omega_{it} \cdot f\left(k_{it}^*; z_{it}\right) - \left(1-\omega_{it}\right) \cdot \left[\left(1-\delta\right) k_{it}^* - \phi_i k_{it}^*\right]\right\} di$$

$$\text{s.t.} \qquad K_t = \int_0^1 k_{it}^* di$$

Aggregate economy and welfare: inner problem

▷ back

- Redistribute $\{k_{i,t+1}\}_i$ taking exit decisions $\{\mathcal{P}^{DE}_{i,t+1}\}_{i\in[0,1]}$ and K^{DE}_{t+1} as given

$$\max_{\left\{k^*_{i,t+1}\right\}_i} \quad \int_0^1 \mathbb{E}_t \left[\mathcal{P}^{DE}_{i,t+1}\left(f(k^*_{i,t+1}, z_{i,t+1}) + (1-\delta)k^*_{i,t+1}\right) + (1-\mathcal{P}^{DE}_{i,t+1}) \cdot \phi_i k^*_{i,t+1}\right] \mathrm{d}i$$

$$\text{s.t.} \qquad \int_0^1 k^*_{i,t+1} \mathrm{d}i = K^{DE}_{t+1}$$

- Lower bound on full misallocation

Misallocation formula: proof

▷ back

- Formally, planner's problem is now the same as solving $Y = \max_{\{k_i\}_i} \int_0^1 f_i(k_i)\mathrm{d}i$, where $f_i(k_i)$ is now expected output

- Apply Hughes and Majerovitz (2024), noting $\frac{dY}{dk} = r^{social} + \delta$

$$\log\left(Y^*/Y^{DE}\right) \approx \frac{1}{2} \cdot \mathcal{E} \cdot \log\left(1 + \frac{\mathrm{Var}\left(r^{social}\right)}{\left(\mathbb{E}\left[r^{social}\right] + \delta\right)^2}\right)$$

- $\mathcal{E}$ is (negative) elasticity of output w.r.t. cost of capital $(r^{social} + \delta)$

Calibration of $\mathcal{E}$

▷ back

- $\mathcal{E}_i$ is the elasticity of expected output with respect to the cost of capital

- Assume that $f(k,z) = z \cdot k^{\alpha}$ and there is no default, then

$$\mathcal{E} = \frac{\alpha}{1-\alpha}$$

- $\alpha = \frac{1}{3}$ implies $\mathcal{E} = \frac{1}{2}$

Summary Statistics

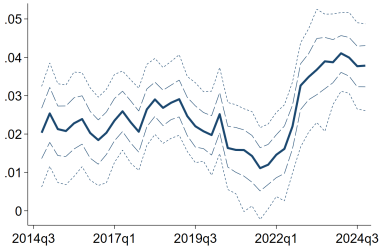
▷ back

	Mean	St. Dev.	p10	p50	p90
Interest rate	4.18	1.69	2.21	3.94	6.60
Maturity (yrs)	6.83	4.65	3.00	5.00	10.25
Real interest rate	2.39	1.24	0.88	2.33	4.00
Prob. Default (%)	1.45	2.53	0.19	0.85	2.88
LGD (%)	34.41	13.17	16.00	35.60	50.00
Loan amount (M)	10.75	67.58	1.11	2.57	22.92
Sales (M)	1,269.93	6,051.48	2.16	58.50	1,560.10
Assets (M)	1,760.37	8,894.15	1.07	35.55	1,782.22
Leverage (%)	72.17	24.68	42.68	71.29	100.00
Return on assets (%)	27.60	58.51	4.56	15.76	47.81
N Loans	65,284				
N Firms	38,751				
N Fixed Rate	32,592				
N Variable Rate	32,692				

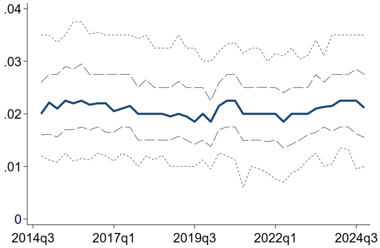
Time series for averages: real interest rate, PD, LGD

▷ back

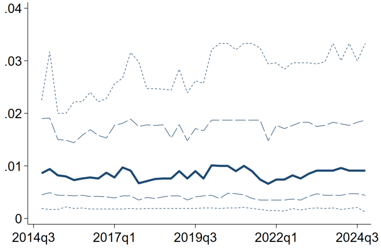
Real interest rate



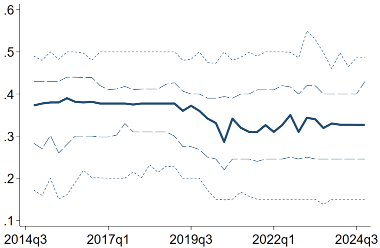
Interest rate spread (var.)



Probability of default



Loss given default



Data cleaning and sample construction

▷ back

We use FR Y-14Q Schedule H.1 data from 2014Q4 to 2024Q4.

Borrower Filters:

- Drop loans without a Tax ID
- Keep only Commercial & Industrial loans to nonfinancial U.S. addresses
- Drop borrowers with NAICS codes:
  - 52 (Finance and Insurance), 92 (Public Administration)
  - 5312 (Real Estate Agents), 551111 (Bank Holding Companies)

Data cleaning and sample construction, cont'd

▷ back

Loan Filters:

- Drop loans with:
  - Negative committed exposure
  - Utilized exposure exceeding committed exposure
  - Origination after or maturity before report date
- Keep only "vanilla" term loans (Facility type $= 7$)
- Drop loans with:
  - Mixed-interest rate structures
  - Maturity less than 1 year or longer than 10 years
  - Implausible interest rates or spreads (outside 1st - 99th percentile)
  - Missing or invalid PD/LGD values (outside $[0, 1]$)
  - PD $= 1$ (flagged as in default)

Forward interest rate expectations

▷ back

To estimate ρ_i for floating rate loans, need estimates of $\mathbb{E}_0[r_t] + s_i$

- Floating rate loans charge reference rate + spread
- Approximate LIBOR/SOFR using Treasury forward yield curve estimates (Gürkaynak et al., 2007)
- Average spread between SOFR and Treasury rates 2018-2025 $\simeq$ 2 basis points
- Assume expectations hypothesis: long rates reflect expected short rates
- Back out $\mathbb{E}_0[r_t] + s_i$ for each loan, using treasury forward rate plus loan's spread

Firm cost of capital: model

▷ back

$$Q_t = \frac{\mathbb{E}_t \left[\mathcal{P}_{t+1} \left(\theta + (1-\theta) \, Q_{t+1} \right) + (1 - \mathcal{P}_{t+1}) \, \phi k_{t+1} / b_{t+1} \right]}{1+\rho}$$

Note that

$$\begin{aligned}
Q_t &= Q_t^P + Q_t^D \\
Q_t^P &= \frac{\mathbb{E}_t \left[\mathcal{P}_{t+1} \left(\theta + (1-\theta) \, Q_{t+1} \right) \right]}{1+\rho} \\
Q_t^D &= \frac{\mathbb{E}_t \left[(1 - \mathcal{P}_{t+1}) \, \phi k_{t+1} / b_{t+1} \right]}{1+\rho}
\end{aligned}$$

That is, we strip the bond into the payment in repay $\left(Q_t^P\right)$ and the payment in default $\left(Q_t^D\right)$. Then:

$$\Lambda = \frac{\mathbb{E}_t \left[(1 - \mathcal{P}_{t+1}) \, \phi k_{t+1} / b_{t+1} \right]}{\mathbb{E}_t \left[\mathcal{P}_{t+1} \left(\theta + (1-\theta) \, Q_{t+1} \right) \right]} = \frac{Q_t^D}{Q_t^P}$$

Wedge due to financial friction

▷ back

$$(lev_{i,t} - \mathcal{M}_{i,t}) \cdot (1 - P_{i,t}) \cdot (1 - LGD_{i,t})$$

- **Lenders** care about average recovery per dollar of debt: $\phi_i(k_i)/b_i = \mathcal{M}_i(1 - LGD_i)$
- **Planner** cares about the marginal recovery: $\phi_i'(k_i) = (1 - LGD_i) \times lev_i$
- Coincide when $lev_i = \mathcal{M}_i$

Firm cost of capital: measurement

▷ back

The firm defaults with probability $(1-P)$ and the lender recovers $(1-LGD)$. Hence

$$Q_t^{D,data} = \frac{(1-P)\,(1-LGD)}{1+\rho}$$

For the payment portion notice that at issuance we have the following condition

$$1 = \sum_{s=1}^{T}\left[\frac{P^s\mathbb{E}_t\left[r_{t+s}\right] + P^{s-1}\,(1-P)\,(1-LGD)}{(1+\rho)^s}\right] + \frac{P^T}{(1+\rho)^T}$$

$$1 = \frac{(1-P)\,(1-LGD)}{1+\rho} + P\frac{\mathbb{E}_t\left[r_{t+1}\right]}{1+\rho} + \left(\sum_{s=2}^{T}\left[\frac{P^s\mathbb{E}_t\left[r_{t+s}\right] + P^{s-1}\,(1-P)\,(1-LGD)}{(1+\rho)^s}\right] + \frac{P^T}{(1+\rho)^T}\right)$$

So, we can define $Q_t^{P,data}$ as $1 = Q_t^{P,data} + Q_t^{D,data}$ so $Q_t^{P,data} = 1 - Q_t^{D,data}$. Finally

$$\Lambda^{data} = \frac{Q_t^{D,data}}{Q_t^{P,data}} = \frac{(1-P)\,(1-LGD)}{1+\rho-(1-P)\,(1-LGD)}$$

Decomposing misallocation

▷ back

Counterfactual I: What if all lenders have the same $\bar{\rho}$?

$$1 + r_{social}^{cf,I} = \overline{(1+\rho)\mathcal{M}} + (lev - \mathcal{M}) \cdot PD \cdot (1 - LGD)$$

Heterogeneity in r_{social}^{cf} → Misallocation due to financial frictions

Counterfactual II: what if we equalize financial frictions?

$$1 + r_{social}^{cf,II} = (1+\rho)\,\mathcal{M} + \overline{(lev - \mathcal{M}) \cdot PD \cdot (1 - LGD)}$$

Heterogeneity in r_{social}^{cf} → Misallocation due to heterogeneous cost of capital

Decomposing ρ

▷ back

- The "real yield" is the implied $\rho^*_{i,t}$ when $P_{i,t} = 1$

$$1 = \sum_{s=1}^{T_{i,t}} \left[\frac{\mathbb{E}_t\left(r_{i,t,s}\right)}{\left(1+\rho^*_{i,t}\right)^s \cdot \mathbb{E}_t(\Pi_{t,s})} \right] + \frac{1}{\left(1+\rho^*_{i,t}\right)^{T_{i,t}} \cdot \mathbb{E}_t(\Pi_{t,T_{i,t}})}$$

- Real yield independent of $P_{i,t}, LGD_{i,t}$

- Only affected by losses through the contractual rate r

Estimates for lender discount rate, firm and social cost of capital

▷ back

	Mean	SD	p10	p50	p90
ρ (%)	1.87	1.55	0.41	1.88	3.62
r^{firm} (%)	0.92	2.80	-0.86	1.26	3.03
r^{social} (%)	1.66	1.78	0.12	1.73	3.47

- Financial frictions/recovery: $\mathbb{E}\left[r_{i,t}^{firm}\right] < \mathbb{E}\left[r_{i,t}^{social}\right], \mathbb{E}\left[\rho_{i,t}\right]$

Variance decomposition

▷ back

	Time	Bank	Firm	Loan
Contractual rate	69	2	15	15
Real rate	49	4	25	22
ρ	43	4	23	30
r^{firm}	17	4	31	49
r^{social}	35	4	25	36

Notes: 1,844 firms and 16,088 loans. Sample restricted to firms with at least five securities.

Within-period dispersion of r^{social}:

- Bank 6%
- Firm 38%
- Loan 55%

Estimating $\mathcal{M}$

▷ back

$$\mathcal{M} = \frac{1 - \gamma \times \frac{Qb'}{k'} \times \frac{\partial \log Q}{\partial \log k'}}{1 + \gamma \times \times \frac{\partial \log Q}{\partial \log b'}}$$

Need Q, γ, and firm leverage Qb'/k' to compute $\mathcal{M}$

1. To compute Q, assume that loans are perpetuities that decay at a geometric rate θ, discounted at the loan's real interest rate r:

   $$Q = \frac{\theta + (1-\theta)Q}{1+r} = \frac{\theta}{r+\theta}$$

   r is directly observed in the data, and we can approximate $\theta = 1/T$
2. Guess a functional approximation $Q(z, k, b, \rho)$
3. Estimate $\log \hat{Q}(z, k, b, \rho)$ for every loan origination; compute partial derivatives
4. At steady state, $\gamma = \theta = 1/T$

Estimating $\mathcal{M}$: Q elasticities

▷ back

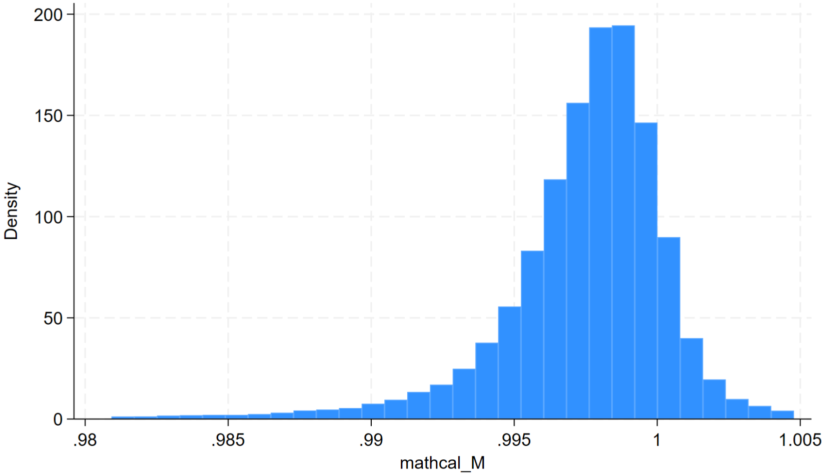
- We approximate (the log of) Q as a polynomial of firm capital, borrowing, productivity and ρ

$$
\begin{aligned}
\log Q_i = \alpha + \beta_k \log k_i + \beta_b \log b_i + \beta_z \log z_i + \beta_\rho \rho_i \\
+\beta_{k,k}(\log k_i)^2 + \beta_{k,b} \log k_i \times \log b_i + \beta_{k,z} \log k_i \times \log z_i + \beta_{k,\rho} \log k_i \times \rho_i \\
+\beta_{b,b}(\log b_i)^2 + \beta_{b,z} \log b_i \times \log z_i + \beta_{b,\rho} \log b_i \times \rho_i \\
+\beta_{z,z}(\log z_i)^2 + \beta_{z,\rho} \log z_i \times \rho_i + \beta_{\rho,\rho}(\rho_i)^2 + \epsilon_i
\end{aligned}
$$

- Capital: tangible assets
- Borrowing: total debt owed by the firm at loan origination
- Productivity: sales over tangible assets
- This allows us to compute $\frac{\partial \log Q}{\partial \log k'}$ and $\frac{\partial \log Q}{\partial \log b'}$

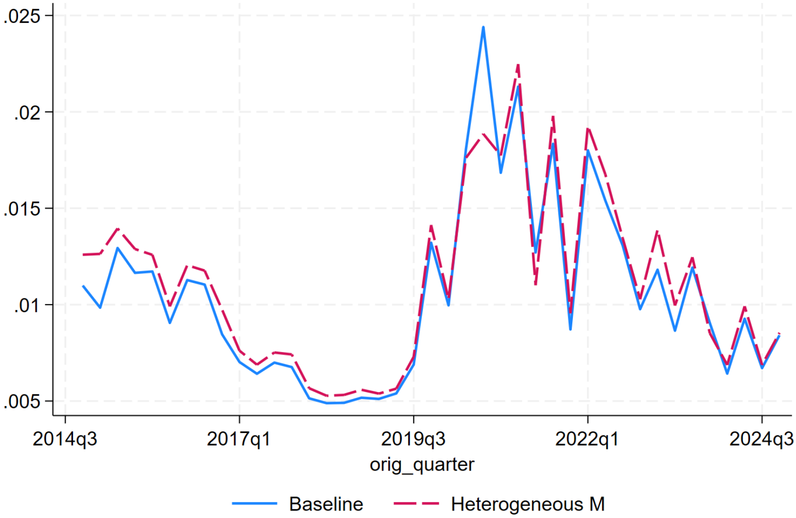
Estimating $\mathcal{M}$: results

▷ back



Misallocation with heterogeneous $\mathcal{M}$

▷ back



r^{social} correlates with standard measures of ARPK

▷ back

	(1)	(2)	(3)	(4)	(5)
	$\log(ARPK)$, Sales	$\log(ARPK)$, EBITDA	$\log(ARPK)$, Sales	$\log(ARPK)$, EBITDA	$\log(ARPK)$, VA
$\log(r^{social} + \delta)$	0.15***	0.24***	0.16**	0.15*	0.39***
	(0.03)	(0.04)	(0.07)	(0.08)	(0.07)
Observations	59294	57334	4184	4072	3432
Adj. R2	0.27	0.22	0.68	0.52	0.61
NAICS4, Quarter FE	yes	yes	yes	yes	yes
Sample	Y-14	Y-14	Compustat	Compustat	Compustat

Robust standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

ARPK-based misallocation

▷ other measures

Focus on Compustat firms to make measures comparable

	$r^{social} + \delta$	$\frac{\text{Sales}}{\text{Capital}}$	$\frac{\text{EBITDA}}{\text{Capital}}$	$\frac{\text{Value Added}}{\text{Capital}}$
$Var(\log)$	0.01	0.19	0.24	0.21
Misallocation (%)	0.36	4.75	6.20	5.28

- Our measure looks only at misallocation coming from heterogeneity in the cost of capital
- ...but does not require detailed data on firm financials (i.e., value added)
- $\implies$ directly applicable to most existing credit registries

Cross-country comparison, approximation

▷ details

	Aleem 1990 Pakistan	Khwaja & Mian 2005 Pakistan	Cavalcanti et al. 2024 Brazil	Beraldi 2025 Mexico	This paper 2025 United States
Years of data	1980–1981	1996–2002	2006–2016	2003–2022	2014–2024
Mean real rate, %	66.8	8.00	83.0	12.4	1.4
SD real rate, %	38.1	2.9	93.3	5.2	1.2
Mean def. prob., %	2.7	16.9	4.0	8.9	1.5
Mean recovery rate, %	42.8	42.8	18.2	63.9	66.6
Implied misallocation, %	6.5	13.5	21.5	2.8	0.8

- Developing countries: higher mean and standard deviation of real interest rates
- U.S.: lower mean and standard deviation of interest rates, higher recovery
- Brazil: most extreme misallocation: 21.5%.

ARPK-based misallocation measures

▷ back

	(1) log $ARPK$, Sales	(2) log $ARPK$, EBITDA	(3) log $ARPK$, Sales	(4) log $ARPK$, EBITDA	(5) log $ARPK$, VA
$\log(r^{social} + \delta)$	0.15***	0.24***	0.16**	0.15*	0.39***
	(0.03)	(0.04)	(0.07)	(0.08)	(0.07)
Observations	59294	57334	4184	4072	3432
Adj. R2	0.27	0.22	0.68	0.52	0.61
NAICS4, Quarter FE	yes	yes	yes	yes	yes
Sample	Y-14	Y-14	Compustat	Compustat	Compustat
Var($\log ARPK$)	1.97	1.52	0.19	0.24	0.21
Misalloc., $ARPK$, %	63.63	46.08	4.75	6.20	5.28
Var($\log(r^{social} + \delta)$)	0.04	0.04	0.01	0.01	0.01
Misalloc., $r^{social} + \delta$, %	0.96	0.96	0.36	0.36	0.36

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Details on cross-country comparison

▷ back to loan pricing ▷ back to cross-country

- For a fixed real interest rate $r_{i,t}$, ρ has a closed-form:

$$1 + \rho_{i,t} = P_{i,t}\left(1 + r_{i,t}\right) + \left(1 - P_{i,t}\right)\left(1 - LGD_{i,t}\right)$$

- Assume all loans have the same maturity:
  1. Obtain mean real rate by subtracting average realized inflation from mean nominal rate
  2. Inflation should not affect standard deviation of nominal rates (or spreads)
- Assume all loans have the same $P_{i,t}, LGD_{i,t}$, equal to the average
- Recovery rates and inflation rates from the World Bank
- Approximate $r_{i,t}^{social} \simeq \rho_{i,t}$ and compute misallocation using our formula:

$$\log(Y_t^*/Y_t^{DE}) = \frac{1}{2}\mathcal{E}\log\left(1 + \frac{Var(\rho_{i,t})}{(\mathbb{E}[\rho_{i,t}] + \delta)^2}\right)$$