Liquidity and Investment in General Equilibrium

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November 2, 2023

CFAR Finance Brown Bag

Liquidity and investment

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- price an exogenous dividend stream, a *Lucas tree*
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This paper:

- How do trading frictions affect firms' investment and the aggregate economy?
  - Why would they matter? Affect owners' discount factor

Liquidity and investment in general equilibrium

Model

- Aiyagari production economy with liquid and illiquid assets in general equilibrium
- Firms take into account that ownership shares trade in frictional asset markets

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    - the discount factor of firms is as if firms have quasi-hyperbolic discounting
    - result from frictions in financial markets
    - present-bias is the empirically relevant case

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   - with commitment: increase in capital with larger trading frictions
   - without commitment: decrease in capital with larger trading frictions

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2. **Quantitative:** trading frictions and aggregate distortions
   - with commitment: increase in capital with larger trading frictions
   - without commitment: decrease in capital with larger trading frictions
3. **Data:** rationalize facts on the cross-section of liquidity and investment

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Aiyagari production economy with liquid and illiquid assets

Households

- idiosyncratic labor risk h
- incomplete markets:
  - liquid bond b, borrowing limit $b \geq \underline{b}$
  - illiquid stock θ, transaction costs $\mathcal{T}$

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Firms

- DRS technology $y = \left(h^{\gamma} k^{1-\gamma}\right)^{\psi}$
- capital accumulation $k_{t+1} = i_t + (1-\delta)k_t \longleftarrow$ firms solve a **dynamic** problem

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Stationary equilibrium

- interest rate r, stock price q, and wage w such that markets clear:

$$\mathbb{E}[b] = 0 \qquad \mathbb{E}[\theta] = 1 \qquad \mathbb{E}[h] = H$$

Household problem

$$\max_{\{c_t, b_{t+1}, \Delta_t^+, \Delta_t^-\}_{t \geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$c_t + q\Delta_t^+ + \frac{b_{t+1}}{1+r} \leq wh_t + d\theta_t + \left(\Delta_t^- - \mathcal{T}(\Delta_t^-)\right) q + b_t$$

$$\theta_{t+1} = \theta_t + \Delta_t^+ - \Delta_t^-$$

$$\Delta_t^- \leq \theta_t \leftarrow \text{ short-selling constraint}$$

$$b_{t+1} \geq \underline{b} \leftarrow \text{ borrowing constraint}$$

$$\mathcal{T}(\Delta_t^-) = \frac{\phi}{2}(\Delta_t^-)^2 \leftarrow \text{ quadratic costs for sellers (e.g., Heaton Lucas 96)}$$

$$\Delta_t^+, \Delta_t^- \geq 0$$

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$$\mathcal{P}^i_{t,t+z} = \frac{1}{u'(c^i_t)} \frac{\partial V^i_t\left(\{k_{t+s}\}_{s\geq 1}\right)}{\partial k_{t+z}}$$

$$= \mathbb{E}_t\left[\sum_{j=0}^{\infty}\left[\beta^j \frac{u'\left(c^i_{t+j}\right)}{u'\left(c^i_t\right)}\left(\theta^i_{t+j}\underbrace{\frac{\partial d_{t+j}}{\partial k_{t+z}}}_{\text{dividends}} + \left(\Delta^{i,-}_{t+j} - \frac{\phi}{2}(\Delta^{i,-}_{t+j})^2 - \Delta^{i,+}_{t+j}\right)\underbrace{\left(\frac{\partial q_{t+j}}{\partial k_{t+z}}\right)^i}_{\text{valuation}}\right)\right]\right]$$

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Effect of a change in capital in:

1. dividends, $\frac{\partial d_{t+j}}{\partial k_{t+z}}$, determined by technological factors
2. stock price, $\left(\frac{\partial q_{t+j}}{\partial k_{t+z}}\right)^i$, agents might disagree
   - follow Grossman and Hart 79: $\left(\frac{\partial q_{t+j}}{\partial k_{t+z}}\right)^i$ represents household i's perception

Marginal propensity to pay: current and future impacts

Lemma: *$\mathcal{P}$ can be decomposed in two terms:*

1. *impact in current wealth*

$$\theta_t^i \left(\frac{\partial d_t}{\partial k_{t+z}} + \left(1 - \frac{\phi}{2}\Delta_{i,t}^{-}\right) \left(\frac{\partial q_t}{\partial k_{t+z}}\right)^i \right)$$

2. *disagreements about future valuations*

$$\mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j}^i \theta_{t+j+1}^i \left[\underbrace{\frac{\frac{\partial d_{t+j+1}}{\partial k_{t+z}} + \left(1 - \Phi_{t+j}^i\right)\left(\frac{\partial q_{t+j+1}}{\partial k_{t+z}}\right)^i}{1 + r_{t+j,t+j+1}^i}}_{\text{benefits of holding the stock at } j} - \underbrace{\left(1 - \frac{\phi}{2}\Delta_{i,t+j}^{-}\right)\left(\frac{\partial q_{t+j}}{\partial k_{t+z}}\right)^i}_{\text{perception of value at } j} \right] \right]$$

$r_{t+j,t+j+1}^i \equiv \frac{1}{\beta} \frac{u'\left(c_{t+j}^i\right)}{\mathbb{E}_{t+j}\left[u'\left(c_{t+j+1}^i\right)\right]} - 1$, $M_{t,t+j}^i = \left(\prod_{u=0}^{j-1} \frac{1}{1+r_{t+u,t+u+1}^i}\right)$, $\Phi_{t+j}^i \equiv \frac{\phi}{2} \frac{\mathbb{E}_{t+j}\left[u'\left(c_{t+j+1}^i\right)\Delta_{i,t+j+1}^{-}\right]}{\mathbb{E}_{t+j}\left[u'\left(c_{t+j+1}^i\right)\right]}$

Competitive perceptions

Assumption [competitive perceptions]: *households' believe they would not benefit from disagreements about future valuations (as in Grossman and Hart 79). Households' marginal propensity to pay simplifies to impact in current wealth.*

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Firm's problem: the manager can transfer income from those shareholders who favor the change in investment ($\mathcal{P}^i_{t,t+s} > 0$) to those who do not favor it ($\mathcal{P}^i_{t,t+s} < 0$). Choose an investment plan such that

$$\int_{\theta,b,h} \mathcal{P}_{t,t+z}(\theta, b, h) \;\; d\Gamma_t(\theta, b, h) = 0$$

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What is the firm's perception about $\frac{\partial q_t}{\partial k_{t+z}}$?

- frictionless case, $\phi = 0$: $\frac{\partial q_t}{\partial k_{t+z}} = \sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^j \frac{\partial d_{t+j}}{\partial k_{t+z}}$ ▷ frictionless case
- competitive perceptions → three-periods model (paper)
- market perceptions

Market perceptions: liquidity premium

- focus on unconstrained buyers: $\Delta_t^- = 0$, $\Delta_t^+ > 0$, $b_{t+1} > \underline{b}$
- bonds' Euler equation: $E_t\left[\beta\frac{u'(c_{t+1})}{u'(c_t)}\right] = \frac{1}{1+r_t}$

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- asset price:

$$q_t = \frac{d_{t+1} + (1 - \mathcal{LP}_t)\, q_{t+1}}{1 + r} \qquad \mathcal{LP}_t \equiv \mathbb{E}_t\left[\phi\Delta_{t+1}^-\right] + \phi\frac{\text{cov}_t\left(u'(c_{t+1}), \Delta_{t+1}^-\right)}{\mathbb{E}_t\left[u'(c_{t+1})\right]}$$

$\mathcal{LP}$ captures liquidity frictions:

- expected marginal transaction costs, $\phi\Delta_{t+1}^- \rightarrow$ lower asset prices
- if sell in bad times, positive covariance $\rightarrow$ further depress asset prices
- define the yield of the stock as $1 + r^\theta \equiv \frac{d_{t+1}+q_{t+1}}{q_t}$, then $\mathcal{LP} = r^\theta - r$

Market perceptions

Assumption [market perceptions]: the firm has the same perceptions as the buyers

$$\left(\frac{\partial q_t}{\partial k_{t+z}}\right)^{market} = \frac{\frac{\partial d_{t+1}}{\partial k_{t+z}} + (1-\Phi^b)\left(\frac{\partial q_{t+1}}{\partial k_{t+z}}\right)^{market}}{1+r} \qquad \text{where } \Phi^b = \frac{\mathcal{LP}}{2}.$$

Iterate forward:

$$\left(\frac{\partial q_t}{\partial k_{t+z}}\right)^{market} = \frac{1}{1-\Phi^b}\sum_{j=1}^{\infty}\left(\frac{1-\Phi^b}{1+r}\right)^j \frac{\partial d_{t+j}}{\partial k_{t+z}}$$

Firm's problem & time inconsistency

Replace market perceptions in firm's problem:

$$\frac{\partial d_t}{\partial k_{t+z}} + \frac{1-\bar{\Phi}}{1-\Phi^b}\sum_{j=1}^{\infty}\left(\frac{1-\Phi^b}{1+r}\right)^j \frac{\partial d_{t+j}}{\partial k_{t+z}} = 0$$

- average transaction cost: $\bar{\Phi} = \frac{\phi}{2}\int_{\theta,b,h}\theta\ \Delta^-\ d\Gamma(\theta,b,h)$
- liquidity premium: $\Phi^b = \frac{\mathcal{LP}}{2}$

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- liquidity premium: $\Phi^b = \frac{\mathcal{LP}}{2}$
- the problem of the firm is time inconsistent iff $\bar{\Phi} \neq \Phi^b$

$$\text{case } z = 1 \qquad \frac{\partial d_t}{\partial k_{t+1}} + \frac{1-\bar{\Phi}}{1+r} \frac{\partial d_{t+1}}{\partial k_{t+1}} = 0$$

$$\text{case } z \geq 2 \qquad \frac{\partial d_{t+z-1}}{\partial k_{t+z}} + \frac{1-\Phi^b}{1+r} \frac{\partial d_{t+z}}{\partial k_{t+z}} = 0$$

Quasi-hyperbolic discounting and time consistency

Quasi-hyperbolic discounting

Proposition: *we can cast the firm's problem as if it has quasi-hyperbolic discounting*

$$V^F(k_t) = \max_{\{k_{t+s}\}_{s\geq 1}} F(k_t, k_{t+1}) + \tilde{\beta} \sum_{s=1}^{\infty} \tilde{\delta}^s F(k_{t+s}, k_{t+s+1})$$

where

$$\tilde{\delta} = \frac{1 - \Phi^b}{1 + r} \qquad \tilde{\beta} = \frac{1 - \bar{\Phi}}{1 - \Phi^b}$$

- quasi-hyperbolic discounting iff $\Phi^b \neq \bar{\Phi}$
- present bias ($\tilde{\beta} < 1$) iff $\bar{\Phi} > \Phi^b$

▷ static labor choice

Direction and magnitude of bias

Proposition: *the difference $\Phi^b - \bar{\Phi}$ is equal to persistence and risk premium effects:*

$$\Phi^b - \bar{\Phi} = \underbrace{\frac{\phi}{2}\left(\tilde{\mathbb{E}}\left[\mathbb{E}_t\left[\Delta^-_{t+1}\right] \middle\| \text{buyer}\right] - \tilde{\mathbb{E}}\left[\mathbb{E}_t\left[\Delta^-_{t+1}\right]\right]\right)}_{\text{persistence effect}} + \underbrace{\frac{\phi}{2}\tilde{\mathbb{E}}\left[\frac{\text{cov}_t\left(u'\left(c_{t+1}\right), \Delta^-_{t+1}\right)}{\mathbb{E}_t\left[u'\left(c_{t+1}\right)\right]} \middle\| \text{buyer}\right]}_{\text{risk premium}}$$

$\tilde{\mathbb{E}}$ is the cross-sectional expectation, weighted by stock shares θ'

no transaction costs: If $\phi = 0$ then $\Phi^b = \bar{\Phi} = 0$, so $\tilde{\beta} = 1$, time consistent problem.

Intuition: persistence and risk premium

Persistence effect:

$$\frac{\phi}{2}\left(\tilde{\mathbb{E}}\left[\mathbb{E}_t\left[\Delta_{t+1}^{-}\right] \| \text{buyer}\right] - \tilde{\mathbb{E}}\left[\mathbb{E}_t\left[\Delta_{t+1}^{-}\right]\right]\right)$$

- difference on average transaction costs for buyers and owners
- smaller for buyers than owners $\rightarrow$ negative term

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Risk premium:

$$\tilde{\mathbb{E}}\left[\frac{\text{cov}_t\left(u'\left(c_{t+1}\right),\Delta^-_{t+1}\right)}{\mathbb{E}_t\left[u'\left(c_{t+1}\right)\right]}\middle\|\text{buyer}\right]$$

- if sell in bad times $\rightarrow$ positive covariance
- quantitatively the persistence effect dominates, so $\tilde{\beta} < 1$
- the problem is time inconsistent and the firm has present bias

Solution with and without commitment

Solution with and without commitment

With commitment

$$\max_{\{k_{t+s}\}_{s\geq 1}} F(k_t, k_{t+1}) + \tilde{\beta} \sum_{s=1}^{\infty} \tilde{\delta}^s F(k_{t+s}, k_{t+s+1})$$

Steady state capital is

$$k^C = \left(\frac{(1-\gamma)\,\psi \tilde{\delta}}{1 - \tilde{\delta}\,(1-\delta)} H^{\gamma\psi} \right)^{\frac{1}{1-(1-\gamma)\psi}}$$

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Without commitment

- markov perfect equilibrium

$$\max_{k'} F(k, k') + \tilde{\beta}\tilde{\delta} W(k')$$

$$W(k') = F(k', g(k')) + \tilde{\delta} W(g(k'))$$

$$k^N = \left(\frac{(1-\gamma)\,\psi\tilde{\beta}\tilde{\delta}}{1-\tilde{\beta}\tilde{\delta}\,(1-\delta)} H^{\gamma\psi} \right)^{\frac{1}{1-(1-\gamma)\psi}}$$

Incomplete markets, transaction costs, and commitment

1. **Complete markets**
   - $\beta(1+r) = 1$, firms discount at rate $\frac{1}{1+r} = \beta$

2. **Aiyagari 94:** incomplete markets without transactions costs
   - $\tilde{\beta} = 1$, no problems of commitment
   - firms discount at rate $\frac{1}{1+r}$
   - precautionary savings: $\beta(1+r) < 1$, *more capital* than in complete markets

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3. **Transactions costs, with commitment**
   - firms discount at rate $\tilde{\delta} = \frac{1-\Phi^b}{1+r}$
   - $\Phi^b \to$ more discounting, less capital than in Aiyagari 94

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4. **Transactions costs, without commitment**
   - firms discount at rate $\tilde{\beta}\tilde{\delta}$, present bias $\tilde{\beta} < 1$
   - less capital than with commitment: $k^n < k^c$

Caveat: for 3. and 4., in GE, r and Φ also change $\rightarrow$ quantitative evaluation

Quantitative evaluation

Calibration

Three sets of parameters:

1. standard or from the literature
2. income process: assume conservative values, do robustness exercises
3. transaction costs: look at the data, consider different values of ϕ

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Parameter	Value	Source
Discount factor β	0.95	Standard
Risk aversion σ	2.00	Standard
Depreciation δ	0.05	Standard
Production weight on labor γ	0.80	Gavazza et al. (2018)
Returns to scale ψ	0.95	Gavazza et al. (2018)
Borrowing limit $\underline{b}$	1.00	Kaplan et al. (2018)
Labor persistence ρ_h	0.50	Conservative, robustness exercises
Labor st dev σ_h	0.03	Conservative, robustness exercises
Transaction cost ϕ	4.00	Data

Data: relative spreads

- Daily data on ordinary shares traded in NYSE (CRSP), relative spreads:

$$RS_{i,t} = \frac{A_{i,t} - B_{i,t}}{0.5(A_{i,t} + B_{i,t})}$$

- 2000Q1 to 2022Q1 (average of daily data), 3k firms, 124k firm-quarter obs

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Relative Spreads, %

	Mean	St. dev.	p10	p50	p90
2000Q1-2022Q1	3.37	2.35	1.54	2.79	5.72

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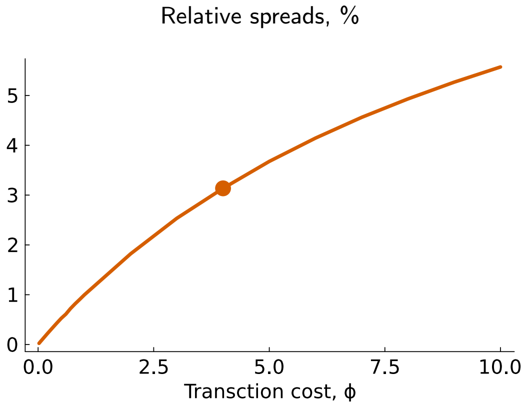
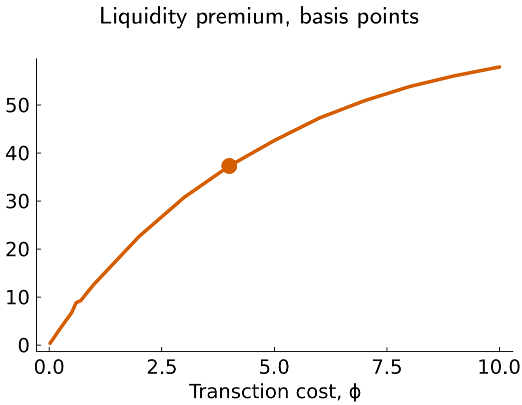
	Mean	St. dev.	p10	p50	p90
2000Q1-2022Q1	3.37	2.35	1.54	2.79	5.72
2000Q1-2006Q1	3.23	2.28	1.57	2.77	5.23
2010Q1-2019Q4	2.93	1.71	1.47	2.52	4.8

consistent with Næs Skjeltorp Ødegaard (2011) and Corwin Schultz (2012)

▷ histogram ▷ weighted by market cap

Calibration of transaction costs





- benchmark calibration: $\phi = 4.0$
- relative spread of 3.1%, consistent with data
- liquidity premium of 37 basis points

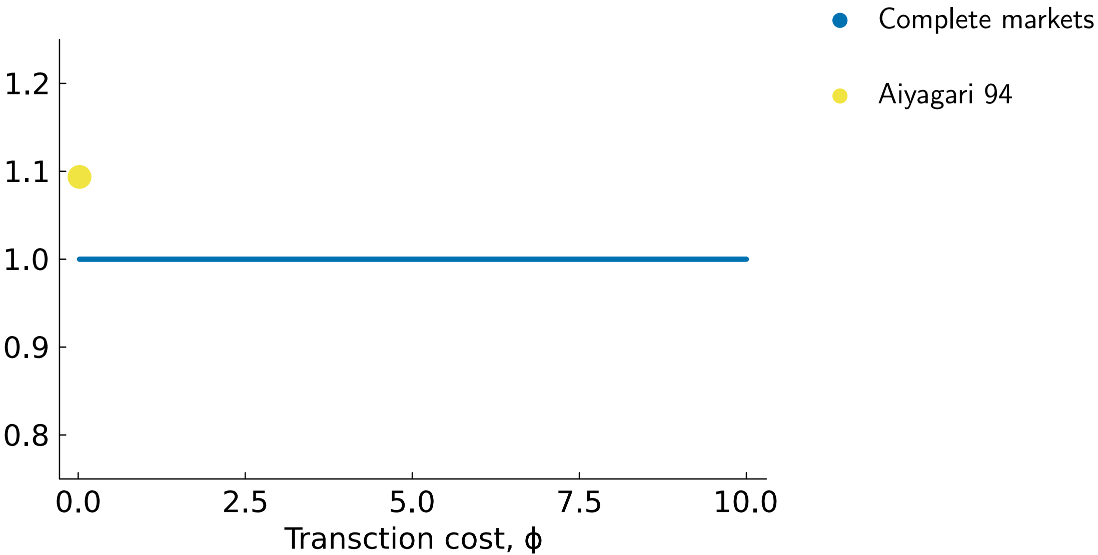
Non targeted moments

	Model	Data
Var log consumption / var log income	0.2	0.3
Mean illiquid assets to GDP	3.4	2.9
Mean liquid assets ($b > 0$) to GDP	0.5	0.23
Share with $b < 0$	0.5	0.2

consumption and income data from Krueger and Perri (2006). Asset data from SCF 2004 (see Kaplan et al., 2018).

consistent with non-targeted moments despite being an stylized model without many quantitative add-ons.

Capital, relative to complete markets



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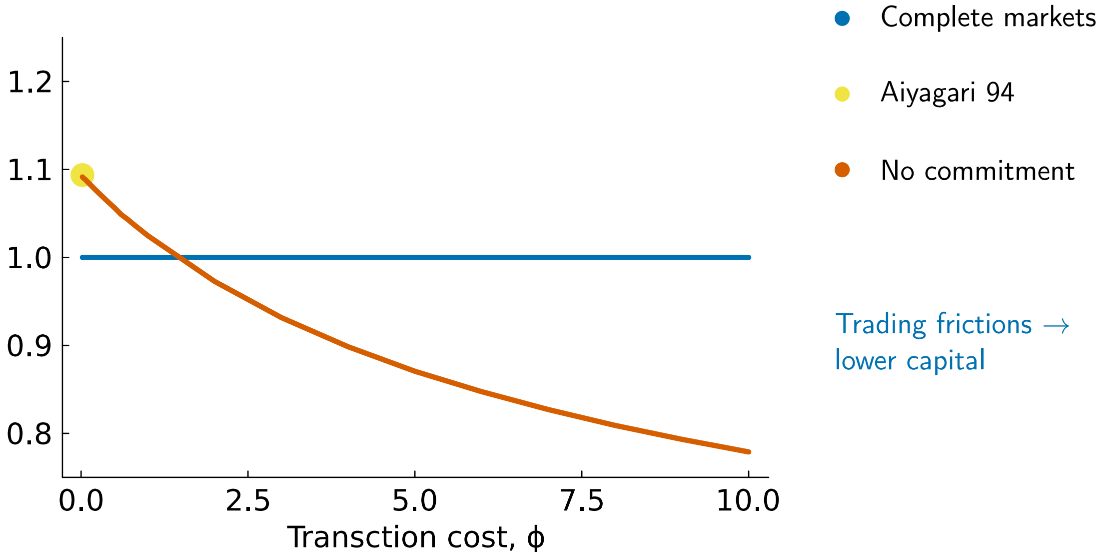

Trading frictions → lower capital

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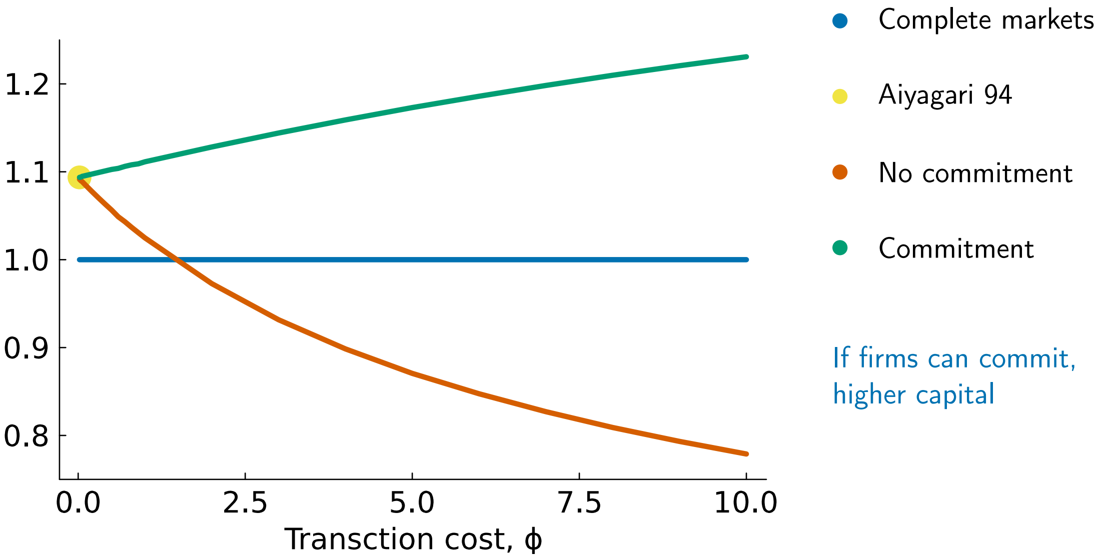

If firms can commit, higher capital

Transmission of trading frictions to investment depends on commitment

With commitment

- trading frictions depress asset prices $\rightarrow$ lower level of capital
- higher precautionary motive for saving $\rightarrow$ larger level of capital
- quantitatively: moderate increase in capital

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Without commitment

- present bias: strong force towards more discounting and lower capital

▷ How does the model work?

Extensions & applications

Corporate bonds

Firms can borrow at interest rate $1 + r^{cb} = \frac{1+r}{1-\tilde{\phi}}$ up to a limit

- If $\tilde{\phi} < \Phi^b$ the firm always borrows to the limit independently of its commitment.
- If $\Phi^b < \tilde{\phi} < \overline{\Phi}$ only the firm without commitment borrows up to the limit.

Corporate bonds

Firms can borrow at interest rate $1 + r^{cb} = \frac{1+r}{1-\tilde{\phi}}$ up to a limit

- If $\tilde{\phi} < \Phi^b$ the firm always borrows to the limit independently of its commitment.
- If $\Phi^b < \tilde{\phi} < \overline{\Phi}$ only the firm without commitment borrows up to the limit.

Implications:

- can alter financing but not investment and the time-inconsistency problem
- firms borrow even if bonds are more illiquid than stocks due to present bias
- rationalize corporate debt that does not rely on the tax advantage of debt

Liquidity & investment in the cross-section

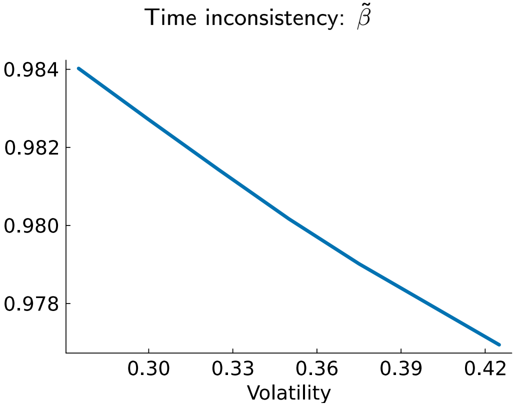
- **Data:** liquid firms invest more than illiquid ones in the cross-section of US public firms (Amihud and Levi, 22)

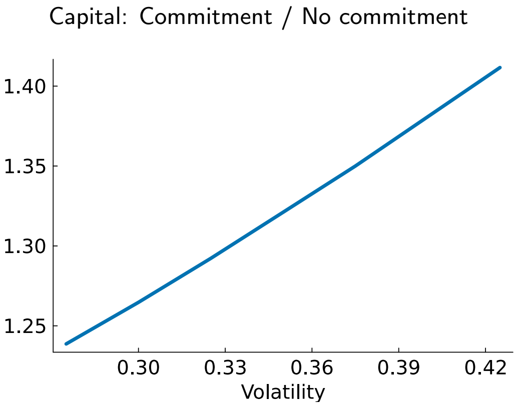
- **Model:** extension with two type of firms, liquid and illiquid ones

Liquidity & investment in the cross-section

- **Data:** liquid firms invest more than illiquid ones in the cross-section of US public firms (Amihud and Levi, 22)
- **Model:** extension with two type of firms, liquid and illiquid ones
- the liquid firm discounts at rate $\frac{1}{1+r}$ with standard exponential discounting
- the discount factor of illiquid firms is $\frac{1-\bar{\Phi}}{1+r}$
- liquid firms invest more than illiquid ones, consistent with the data

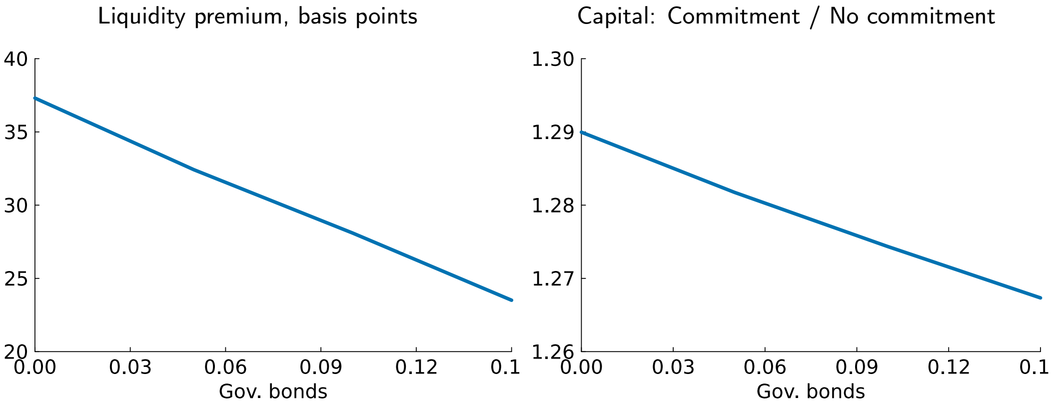
Demand of liquidity: increase idiosyncratic volatility





- With commitment: more precautionary savings $\rightarrow$ more capital
- Without commitment: more time inconsistency $\rightarrow$ less capital

Supply of liquidity & government bonds



- Capital closer to complete markets
- With commitment: less precautionary savings → less capital
- Without commitment: less time inconsistency → more capital

Short-termism

Evidence on short-termism:

- an excessive focus on short-term results at the expense of long-term interests (Graham et al. 05, Terry 22, Fink 15)
- public firms distort their investment to meet short-term targets (Graham et al., 05).

Model: short-termism as a result of (i) trading frictions, and (ii) lack of commitment.

Conclusions

- Aiyagari production economy, with liquid and illiquid assets in general equilibrium
- The problem of the firm is time inconsistent
  - result from frictions in financial markets
  - the discount factor of firms is as if they have quasi-hyperbolic discounting
- Aggregate distortions due to trading frictions depend on commitment
- Rationalize empirical regularities on liquidity and investment

Appendix

Related Literature

- Incomplete markets & firm insurance: Diamond (1967), Dreze (1974), **Grossman Hart (1979)**, Aiyagari Gertler (1991), Heaton Lucas (1996), Magill Quinzii (1996), Espino Kozlowski Sanchez (2018)
  New: Trading frictions and/or GE

- Illiquid assets & macro: Kaplan Violante (2014), Cui Radde (2019), Jeenas Lagos (2020)
  New: Dynamic firm's problem with liquidity frictions

- Hyperbolic discounting: Krusell Smith (2003), Azzimonti (2011), Amador (2012), Cao Werning (2018)
  New: Hyperbolic discounting as a result

- Short-termism: Graham Harvey Rajgopal (2005), Terry (2022)
  New: Don't need additional constraints

Frictionless case, $\phi = 0$

Disagreements about future valuations simplifies to

$$\theta^i_{t+j+1}\left(\frac{\frac{\partial d_{t+j+1}}{\partial k_{t+z}} + \left(\frac{\partial q_{t+j+1}}{\partial k_{t+z}}\right)^i}{1 + r^i_{t+j,t+j+1}} - \left(\frac{\partial q_{t+j}}{\partial k_{t+z}}\right)^i\right)$$

Frictionless case, $\phi = 0$

Disagreements about future valuations simplifies to

$$\theta^i_{t+j+1}\left(\frac{\frac{\partial d_{t+j+1}}{\partial k_{t+z}} + \left(\frac{\partial q_{t+j+1}}{\partial k_{t+z}}\right)^i}{1 + r^i_{t+j,t+j+1}} - \left(\frac{\partial q_{t+j}}{\partial k_{t+z}}\right)^i\right)$$

Owners have $\theta^i_{t+j+1} > 0$:

- Not at the borrowing constraint: $r^i_{t+j,t+j+1} = r_{t+j}$
- Compare costs and benefits at the market interest rate
- This implies no disagreement on future valuations
- The marginal propensity to pay depends only on current impact

$$\mathcal{P}^i_{t,t+z} = \theta^i_t\left(\frac{\partial d_t}{\partial k_{t+z}} + \frac{\partial q_t}{\partial k_{t+z}}\right)$$

- Standard problem of the firm: Maximize current value $d + q$

▷ back

Firm: static labor choice

- Static labor choice

$$\max_{l} \left(l^{\gamma} k^{1-\gamma} \right)^{\psi} - wl$$

with labor demand $l = \psi\gamma \frac{y}{w}$

- In equilibrium $w = \psi\gamma k^{(1-\gamma)\psi}$

- Dividends are

$$d_t = F(k_t, k_{t+1}) = zk_t^{\alpha} + (1-\delta)k_t - k_{t+1}$$

where $z = (1-\gamma\psi)\left(\frac{\gamma\psi}{w}\right)^{\frac{\gamma\psi}{1-\gamma\psi}}$ and $\alpha = \frac{(1-\gamma)\psi}{1-\gamma\psi}$

▷ back

Government bonds

- Introduce government bonds
- Lump-sum taxes to pay for the debt services
- Bonds market clearing

$$\int b'(\theta, b, h) d\Gamma(\theta, b, h) = B^g$$

- As B^g increases: more liquid assets

Public vs private firms

- Asker et al. (2015) finds that public firms invest substantially less than private firms.
- We add private firms to the benchmark equilibrium. Private firms are owned by only one household and are not traded in financial markets.
- The investment decisions of private firms are independent of ϕ, while investment in public firms decreases with the transaction cost.
- For most values of ϕ private firms invest more than public firms, consistent with the empirical evidence.

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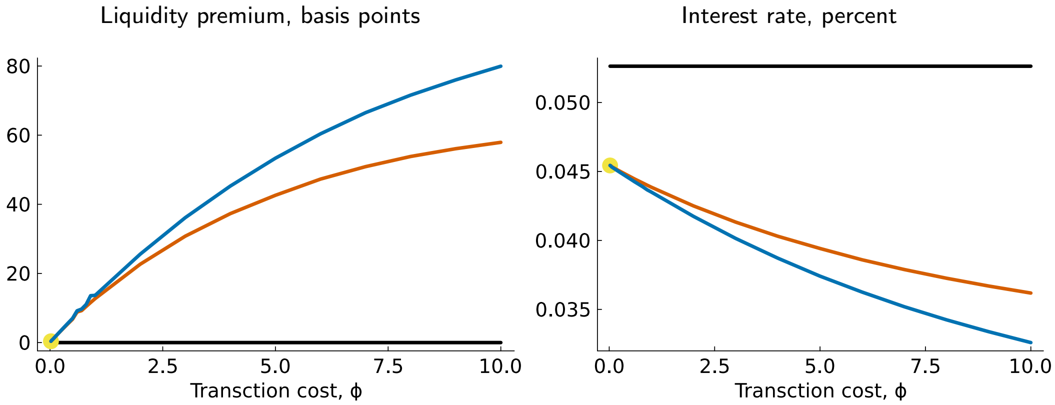
Commitment: constant discounting



- Higher $\phi \rightarrow$ bonds *better* than stocks $\rightarrow$ higher liquidity premium & lower r
- Capital with commitment about constant, recall $\tilde{\delta} = \frac{1-\Phi}{1+r}$

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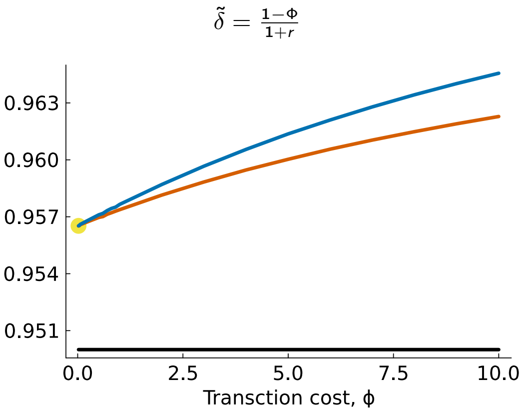
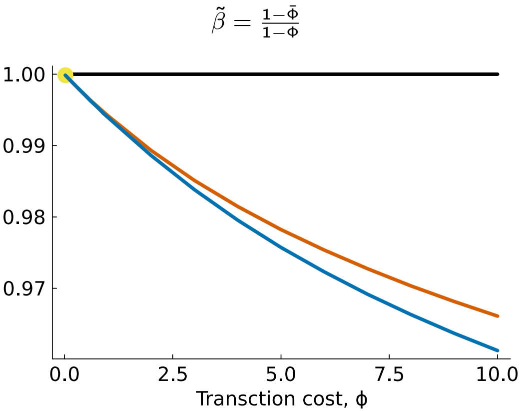
Lack of commitment: quasi-hyperbolic discounting with present bias





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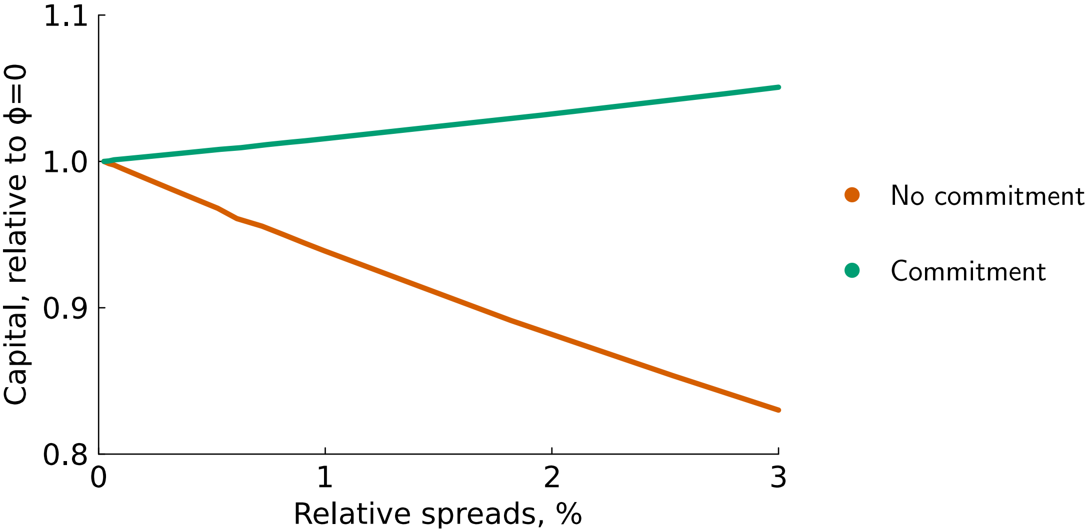
Capital and relative spreads



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Data: relative spreads, weighted by market capitalization

Relative Spreads, %

	Mean	St. dev.	p10	p50	p90
2000Q1-2022Q1	2.31	1.26	1.24	1.98	3.78
2000Q1-2006Q1	2.64	1.27	1.39	2.35	4.23
2010Q1-2019Q4	1.88	0.8	1.15	1.69	2.84

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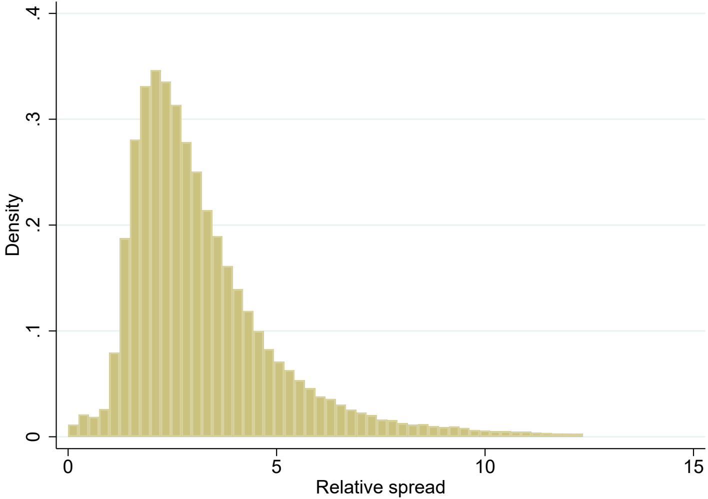
Relative spreads



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