Liquidity and Investment in General Equilibrium

Nicolas Caramp
UC Davis

Julian Kozlowski
St Louis Fed

Keisuke Teeple
U Waterloo

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SED, Cartagena

Liquidity and Investment

Liquidity is important for asset pricing

- Price an exogenous dividend stream, a *Lucas tree*
- E.g., Amihud Mendelson Pedersen (2005)

Liquidity is important for household behavior

- Usually in partial equilibrium and/or firms not affected by trading frictions
- E.g., Kaplan Violante 2014, HANK

This paper:

- How do trading frictions affect firms' investment and the aggregate economy?
  - Why might they matter? Affect owners' discount factor

Liquidity and Investment in General Equilibrium

Model

- Aiyagari production economy with liquid and illiquid assets in general equilibrium
- Firms take into account that ownership shares trade in frictional asset markets

Results

1. **Theory:** The problem of the firm is time inconsistent
   - The discount factor of firms is as if firms have quasi-hyperbolic discounting
   - Result from frictions in financial markets
   - Present-bias is the empirically relevant case
2. **Quantitative:** Trading frictions and aggregate distortions
   - With commitment: Capital does not change with trading frictions
   - Without commitment: Lower capital with larger trading frictions
3. **Data:** Rationalize facts on the cross-section of liquidity and investment

Model

Model

Aiyagari production economy with liquid and illiquid assets

Households

- Idiosyncratic labor risk h
- Incomplete markets:
  - Liquid bond b, borrowing limit $b \geq \underline{b}$
  - Illiquid stock θ, transaction costs $\mathcal{T}$

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Firms

- DRS technology $y = \left(h^{\gamma} k^{1-\gamma}\right)^{\psi}$
- Capital accumulation $k_{t+1} = i_t + (1-\delta)k_t \longleftarrow$ firms solve a **dynamic** problem

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- Owners: Households, with illiquid stock shares θ

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Stationary Equilibrium

- Interest rate r, stock price q, and wage w such that markets clear:

$$\mathbb{E}[b] = 0 \qquad \mathbb{E}[\theta] = 1 \qquad \mathbb{E}[h] = H$$

Household Problem

$$V(\theta, b, h) = \max_{c, b', \Delta^+, \Delta^-} u(c) + \beta \mathbb{E}\left[V\left(\theta', b', h'\right)\right]$$

subject to

$$c + b' + q\Delta^+ \leq wh + b(1+r) + d\theta + q\left(\Delta^- - \mathcal{T}\left(\Delta^-\right)\right)$$

$$\theta' = \theta + \Delta^+ - \Delta^-$$

$$\Delta^- \leq \theta \leftarrow \text{short-selling constraint}$$

$$b' \geq \underline{b} \leftarrow \text{borrowing constraint}$$

$$\mathcal{T}\left(\Delta^-\right) = \frac{\phi}{2}\left(\Delta^-\right)^2 \leftarrow \text{Transaction costs for sellers (e.g., Heaton Lucas 96)}$$

$$\Delta^+, \Delta^- \geq 0$$

Owners Valuation

- Let $\tilde{q}(\theta, b, h)$ be owners's valuation in units of the consumption good

$$\tilde{q}(\theta, b, h) = \frac{V_\theta(\theta, b, h)}{u'(c)}$$

where V_θ is the marginal valuation of stocks.

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- Owners valuation is

$$\tilde{q}(\theta, b, h) = d + \left(1 - \phi\Delta^-(\theta, b, h)\right) q$$

- Buyers, $\Delta^- = 0$: agree the value of the firm is $\tilde{q}(\theta, b, h) = d + q$
- Sellers: Heterogeneous valuations, depend on marginal transaction cost $\phi\Delta^-$

$\rightarrow$ Disagreement among owners on the valuation of the firm

Firm's Objective

Assumption 1: Firm maximizes owners' valuation weighted by ownership shares.

$$\int_{\theta,b,h} \theta \underbrace{\left[d + (1 - \phi\Delta^{-}(\theta, b, h))q\right]}_{\text{owners' valuation}} d\Gamma(\theta, b, h)$$

In spirit of Grossman Hart 1979 (paper also considers Dreze 1974 and DeMarzo 1993).

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Define $\bar{\Phi}$ as the weighted average marginal transaction cost

$$\bar{\Phi} \equiv \phi \int_{\theta,b,h} \theta\Delta^{-}(\theta, b, h) d\Gamma(\theta, b, h)$$

The firm maximizes

$$d + \left(1 - \bar{\Phi}\right) q$$

The Frictionless Case $\phi = 0$

- The firm's objective is to maximize $d + q$
- The price is equal to $q = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t d_t$
- Standard time-consistent problem
- Maximize the NPV of dividends, discounted at the risk-free rate

▷ Three-period model $\implies$ analytical characterization of time inconsistency

Euler Equation

$$(1-\phi\Delta_t^-)q_t = E_t\left[\beta\frac{u'(c_{t+1})}{u'(c_t)}\right](d_{t+1}+(1-\Phi_t)\,q_{t+1})+\eta_t$$

where η_t is the Lagrange multiplier on $\Delta^- \leq \theta$ and

$$\Phi_t \equiv \mathbb{E}_t\left[\phi\Delta_{t+1}^-\right]+\phi\frac{\text{cov}_t\left(u'(c_{t+1}),\Delta_{t+1}^-\right)}{\mathbb{E}_t\left[u'(c_{t+1})\right]}$$

Φ captures liquidity frictions:

1. Expected marginal transaction costs: $\phi\Delta_{t+1}^- \rightarrow$ lower asset prices
2. Positive covariance if sell in bad times $\rightarrow$ further depress asset prices

Buyers

- Focus on unconstrained buyers: $\Delta_t^- = 0$, $\Delta_t^+ > 0$, $b_{t+1} > \underline{b}$
- Bonds' Euler equation: $E_t\left[\beta\frac{u'(c_{t+1})}{u'(c_t)}\right] = \frac{1}{1+r_t}$

Buyers

- Focus on unconstrained buyers: $\Delta_t^- = 0$, $\Delta_t^+ > 0$, $b_{t+1} > \underline{b}$
- Bonds' Euler equation: $E_t\left[\beta \frac{u'(c_{t+1})}{u'(c_t)}\right] = \frac{1}{1+r_t}$
- Asset price:

$$q_t = \frac{d_{t+1} + (1-\Phi)\, q_{t+1}}{1+r}$$

- **Liquidity premium**
  - Define the yield of the stock as $1 + r^\theta \equiv \frac{d_{t+1}+q_{t+1}}{q_t}$
  - The liquidity premium is $r^\theta - r = \Phi$

Assumption 2: The firm takes $\bar{\Phi}$ and Φ as given.

Firm's Problem

$$V^F(k_t) = \max_{\{k_{t+s}\}_{s\geq 1}} d_t + (1 - \bar{\Phi}) q_t$$

subject to

$$q_t = \frac{d_{t+1} + (1 - \Phi)\, q_{t+1}}{1 + r}$$

where $d_t = F(k_t, k_{t+1}) = z k_t^{\alpha} + (1 - \delta) k_t - k_{t+1}$

▷ static labor choice

Quasi-hyperbolic Discounting and Time Consistency

Quasi-Hyperbolic Discounting

The firm's problem is

$$V^F(k_t) = \max_{\{k_{t+s}\}_{s\geq 1}} F(k_t, k_{t+1}) + \tilde{\beta} \sum_{s=1}^{\infty} \tilde{\delta}^s F(k_{t+s}, k_{t+s+1})$$

where

$$\tilde{\delta} = \frac{1-\Phi}{1+r} \qquad \tilde{\beta} = \frac{1-\bar{\Phi}}{1-\Phi}$$

- Quasi-hyperbolic discounting iff $\Phi \neq \bar{\Phi}$
- Present bias ($\tilde{\beta} < 1$) iff $\bar{\Phi} > \Phi$

Direction and Magnitude of Bias Depends on Φ vs $\overline{\Phi}$

Proposition: The difference $\Phi - \bar{\Phi}$ is equal to persistence and risk premium effects:

$$\Phi - \bar{\Phi} = \underbrace{\phi \left(\tilde{\mathbb{E}} \left[\mathbb{E}_t \left[\Delta^{-}_{t+1} \right] \middle\| \text{buyer} \right] - \tilde{\mathbb{E}} \left[\mathbb{E}_t \left[\Delta^{-}_{t+1} \right] \right] \right)}_{\text{persistence effect}} + \underbrace{\phi \tilde{\mathbb{E}} \left[\frac{\text{cov}_t \left(u' \left(c_{t+1} \right), \Delta^{-}_{t+1} \right)}{\mathbb{E}_t \left[u' \left(c_{t+1} \right) \right]} \middle\| \text{buyer} \right]}_{\text{risk premium}}$$

$\tilde{\mathbb{E}}$ is the cross-sectional expectation, weighted by stock shares θ'

No transaction costs: If $\phi = 0$ then $\Phi = \bar{\Phi} = 0$, so $\tilde{\beta} = 1$, time consistent problem.

Intuition: Persistence and Risk Premium Effects

Persistence effect:

$$\tilde{\mathbb{E}}\left[\mathbb{E}_t\left[\Delta^-_{t+1}\right] \| \text{buyer}\right] - \tilde{\mathbb{E}}\left[\mathbb{E}_t\left[\Delta^-_{t+1}\right]\right]$$

- Difference on transaction costs for buyers and owners
- Expected transaction costs are smaller for buyers than owners $\rightarrow$ negative term

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Risk premium:

$$\tilde{\mathbb{E}}\left[\frac{\text{cov}_t\left(u'\left(c_{t+1}\right), \Delta^-_{t+1}\right)}{\mathbb{E}_t\left[u'\left(c_{t+1}\right)\right]} \middle\| \text{buyer}\right]$$

- If sell in bad times $\rightarrow$ positive covariance
- Quantitatively the persistence effect dominates, $\bar{\Phi} > \Phi$, so $\tilde{\beta} < 1$
- The problem is time inconsistent and the firm has present bias

Solution With and Without Commitment

Solution With and Without Commitment

With commitment

$$\max_{\{k_{t+s}\}_{s\geq 1}} F(k_t, k_{t+1}) + \tilde{\beta} \sum_{s=1}^{\infty} \tilde{\delta}^s F(k_{t+s}, k_{t+s+1})$$

Steady state capital is

$$k^C = \left(\frac{(1-\gamma)\,\psi\tilde{\delta}}{1-\tilde{\delta}\,(1-\delta)} H^{\gamma\psi} \right)^{\frac{1}{1-(1-\gamma)\psi}}$$

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Without commitment

- Markov Perfect Equilibrium

$$\max_{k'} F(k, k') + \tilde{\beta}\tilde{\delta} W(k')$$

$$W(k') = F(k', g(k')) + \tilde{\delta} W(g(k'))$$

$$k^N = \left(\frac{(1-\gamma)\,\psi\tilde{\beta}\tilde{\delta}}{1-\tilde{\beta}\tilde{\delta}\,(1-\delta)} H^{\gamma\psi} \right)^{\frac{1}{1-(1-\gamma)\psi}}$$

Incomplete Markets, Transaction Costs, and Commitment

1. **Complete Markets**
   - $\beta(1+r) = 1$, firms discount at rate $\frac{1}{1+r} = \beta$

2. **Aiyagari 94:** Incomplete markets without transactions costs
   - $\tilde{\beta} = 1$, no problems of commitment
   - Firms discount at rate $\frac{1}{1+r}$
   - Precautionary savings: $\beta(1+r) < 1$, *more capital* than in complete markets

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3. **Transactions costs, with commitment**
   - Firms discount at rate $\tilde{\delta} = \frac{1-\Phi}{1+r}$
   - $\Phi \rightarrow$ more discounting, less capital than in Aiyagari 94

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   - Firms discount at rate $\tilde{\delta}=\frac{1-\Phi}{1+r}$
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4. **Transactions costs, without commitment**
   - Firms discount at rate $\tilde{\beta}\tilde{\delta}$, present bias $\tilde{\beta}<1$
   - Less capital than with commitment: $k^n < k^c$

<ins>Caveat:</ins> For 3. and 4., in GE, r and Φ also change $\rightarrow$ quantitative evaluation

Quantitative Evaluation

Calibration

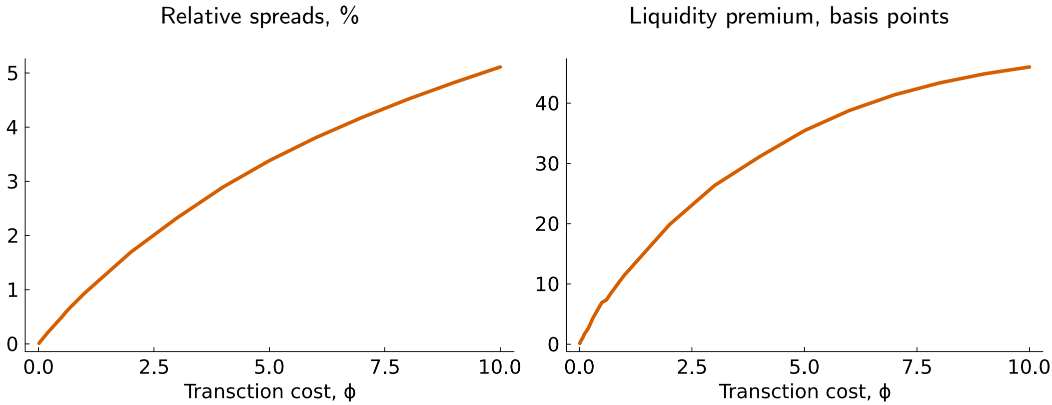
Three sets of parameters:

1. Standard or from the literature
2. Income process: assume conservative values, do robustness exercises
3. Transaction costs: look at the data, consider different values of ϕ ▷ data

Parameter	Value	Source
Discount factor β	0.95	Standard
Risk aversion σ	2.00	Standard
Depreciation δ	0.05	Standard
Production weight on labor γ	0.80	Gavazza et al. (2018)
Returns to scale ψ	0.95	Gavazza et al. (2018)
Borrowing limit $\underline{b}$	1.00	Kaplan et al. (2018)
Labor persistence ρ_h	0.50	Conservative, robustness exercises
Labor st dev σ_h	0.03	Conservative, robustness exercises
Transaction cost ϕ	4.00	Data

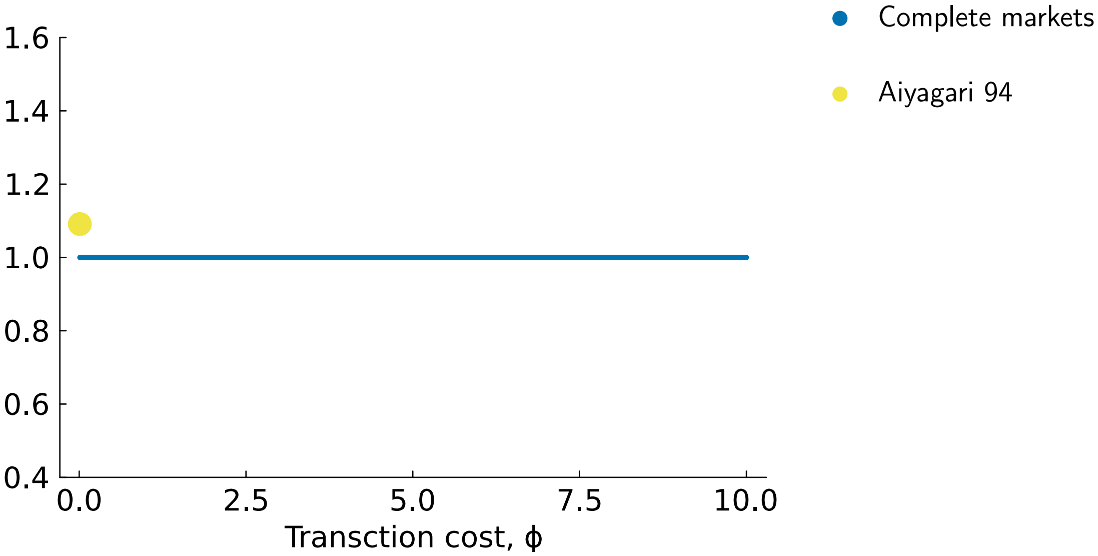
▷ Non-targeted moments

Calibration of transaction costs



- Benchmark calibration: $\phi = 4.0$
- Relative spread of 2.8%, consistent with data
- Liquidity premium of 31 basis points

Capital, relative to complete markets



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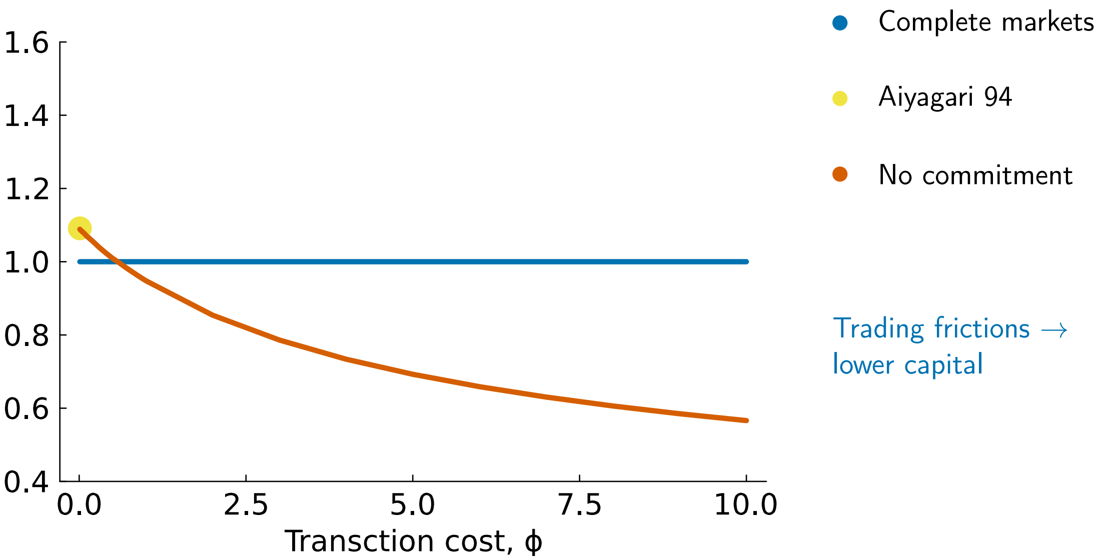
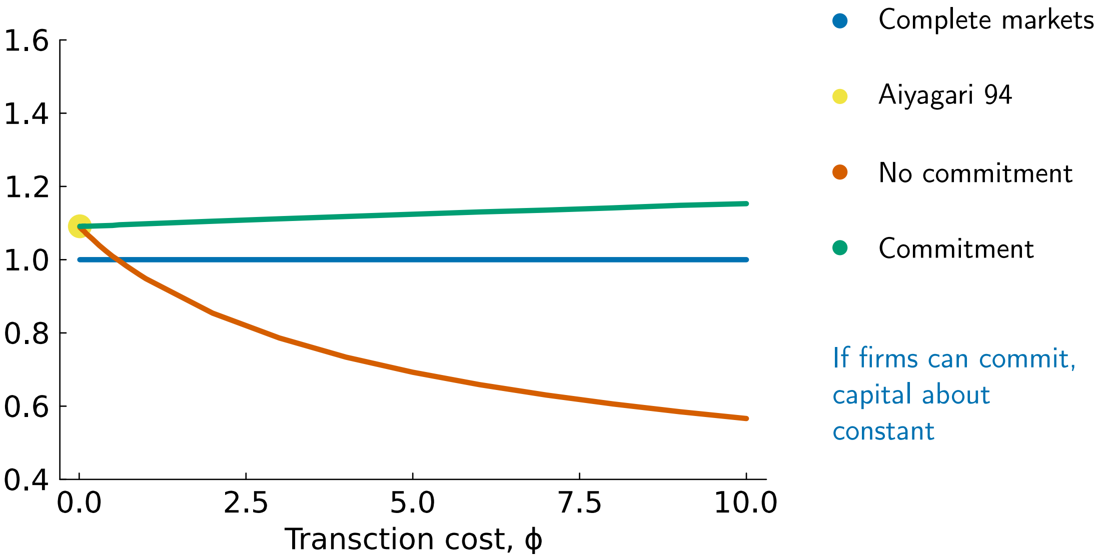


Trading frictions → lower capital

Capital, relative to complete markets



If firms can commit, capital about constant

Transmission of Trading Frictions to Investment Depends on Commitment

With commitment

- Trading frictions depress asset prices $\rightarrow$ lower level of capital
- Higher precautionary motive for saving $\rightarrow$ larger level of capital
- Quantitatively similar, capital does not change significantly

Transmission of Trading Frictions to Investment Depends on Commitment

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Without commitment

- Present bias: strong force towards more discounting and lower capital

▷ How does the model work?

Extensions & Applications

▷ See extensions

- Corporate Bonds
- Liquidity & Investment in the Cross-Section
- Demand of Liquidity: Increase idiosyncartic Volatility
- Liquidity & Investment over the Business Cycle
- Supply of Liquidity & Government Bonds
- Short-termism

Conclusions

- Aiyagari production economy, with liquid and illiquid assets in general equilibrium
- The problem of the firm is time inconsistent
  - Result from frictions in financial markets
  - The discount factor of firms is as if they have quasi-hyperbolic discounting
- Aggregate distortions due to trading frictions depend on commitment
- Rationalize empirical regularities on liquidity and investment

Appendix

Related Literature

- Incomplete markets & firm insurance: Diamond (1967), Dreze (1974), **Grossman Hart (1979)**, Aiyagari Gertler (1991), Heaton Lucas (1996), Magill Quinzii (1996), Espino Kozlowski Sanchez (2018)
  New: Trading frictions and/or GE

- Illiquid assets & macro: Kaplan Violante (2014), Cui Radde (2019), Jeenas Lagos (2020)
  New: Dynamic firm's problem with liquidity frictions

- Hyperbolic discounting: Krusell Smith (2003), Azzimonti (2011), Amador (2012), Cao Werning (2018)
  New: Hyperbolic discounting as a result

- Short-termism: Graham Harvey Rajgopal (2005), Terry (2022)
  New: Don't need additional constraints

Time Inconsistency in a Three-Period Model

Three-Period Model

Simplified model to show the time-consistency problem

- Three periods: $t \in \{0, 1, 2\}$
- No income risk, two type of households with income $\{H, L, H\}$ and $\{L, H, L\}$
- No bonds

Three-period Model: Euler Equations & Firm's Value

Euler equations:

$$\left(1-\phi\Delta_0^{j-}\right) q_0 = \beta\frac{u'\left(c_1^j\right)}{u'\left(c_0^j\right)} d_1 + \beta\frac{u'\left(c_1^j\right)}{u'\left(c_0^j\right)}\left(1-\phi\Delta_1^{j-}\right) q_1$$

$$\left(1-\phi\Delta_1^{j-}\right) q_1 = \beta\frac{u'\left(c_2^j\right)}{u'\left(c_1^j\right)} d_2$$

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$$\left(1-\phi\Delta_1^{j-}\right)q_1=\beta\frac{u'\left(c_2^j\right)}{u'\left(c_1^j\right)}d_2$$

Firm's value:

$$\sum_{j\in\{l,h\}}\frac{\theta_0^j}{2}\left[d_0+(1-\phi\Delta_0^{j-})q_0\right]$$

$$\sum_j\frac{\theta_0^j}{2}\left[d_0+\beta\frac{u'\left(c_1^j\right)}{u'\left(c_0^j\right)}d_1+\beta^2\frac{u'\left(c_2^j\right)}{u'\left(c_0^j\right)}d_2\right]$$

Time Consistency in the Three-Period Model

Problem in period 0

$$\max_{k_1,k_2\geq 0}\sum_j \frac{\theta_0^j}{2}\left[d_0+\beta\frac{u'\left(c_1^j\right)}{u'\left(c_0^j\right)}d_1+\beta^2\frac{u'\left(c_2^j\right)}{u'\left(c_0^j\right)}d_2\right]$$

Problem in period 1

$$\max_{k_2\geq 0}\sum_j \frac{\theta_1^j}{2}\left[d_1+\beta\frac{u'\left(c_2^j\right)}{u'\left(c_1^j\right)}d_2\right]$$

Time Consistency in the Three-Period Model

Problem in period 0

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Problem in period 1

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The problem is time consistent iff the discounting between period 1 and 2 coincides

$$\frac{\sum_j \frac{\theta_0^j}{2}\beta^2\frac{u'(c_2^j)}{u'(c_0^j)}}{\sum_j \frac{\theta_0^j}{2}\beta\frac{u'(c_1^j)}{u'(c_0^j)}}=\sum_j \frac{\theta_1^j}{2}\beta\frac{u'(c_2^j)}{u'(c_1^j)}$$

Three-Period Model, Frictionless Case $\phi = 0$

The Euler equation implies equalization of marginal rates of substitution across agents:

$$\frac{q_t}{d_{t+1} + q_{t+1}} = \beta \frac{u'\left(c^j_{t+1}\right)}{u'\left(c^j_t\right)}$$

Hence

$$\underbrace{\frac{\sum_j \frac{\theta^j_0}{2} \beta^2 \frac{u'(c^j_2)}{u'(c^j_0)}}{\sum_j \frac{\theta^j_0}{2} \beta \frac{u'(c^j_1)}{u'(c^j_0)}}}_{\substack{t=0 \text{ discount between} \\ t=1 \text{ and } t=2}} =$$

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- The problem is time consistent when $\phi = 0$!

Three-periods Model With Trading Frictions, $\phi > 0$

With transaction costs:

$$\frac{\sum_j \frac{\theta_0^j}{2} \beta^2 \frac{u'(c_2^j)}{u'(c_0^j)}}{\sum_j \frac{\theta_0^j}{2} \beta \frac{u'(c_1^j)}{u'(c_0^j)}} \neq \sum_j \frac{\theta_1^j}{2} \beta \frac{u'(c_2^j)}{u'(c_1^j)}$$

- The intertemporal marginal rates of substitution are **not** equalized across agents
- The problem is time inconsistent

▷ Back

Firm: Static labor choice

- Static labor choice

$$\max_{l} \left(l^{\gamma} k^{1-\gamma}\right)^{\psi} - wl$$

with labor demand $l = \psi\gamma\frac{y}{w}$

- In equilibrium $w = \psi\gamma k^{(1-\gamma)\psi}$

- Dividends are

$$d_t = F(k_t, k_{t+1}) = zk_t^{\alpha} + (1-\delta)k_t - k_{t+1}$$

where $z = (1-\gamma\psi)\left(\frac{\gamma\psi}{w}\right)^{\frac{\gamma\psi}{1-\gamma\psi}}$ and $\alpha = \frac{(1-\gamma)\psi}{1-\gamma\psi}$

▷ back

Government Bonds

- Introduce government bonds
- Lump-sum taxes to pay for the debt services
- Bonds market clearing

$$\int b'(\theta, b, h) d\Gamma(\theta, b, h) = B^g$$

- As B^g increases: more liquid assets

Public vs Private Firms

- Asker et al. (2015) finds that public firms invest substantially less than private firms.
- We add private firms to the benchmark equilibrium. Private firms are owned by only one household and are not traded in financial markets.
- The investment decisions of private firms are independent of ϕ, while investment in public firms decreases with the transaction cost.
- For most values of ϕ private firms invest more than public firms, consistent with the empirical evidence.

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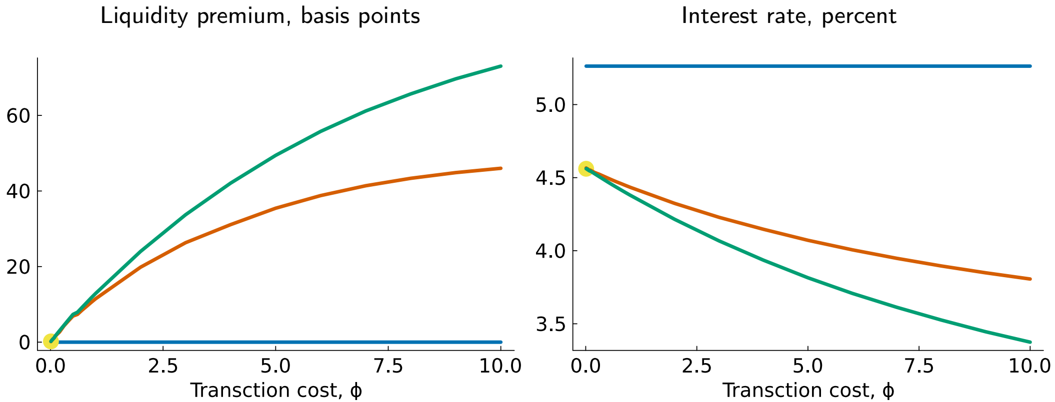
Commitment: Constant discounting



- Higher $\phi \to$ bonds *better* than stocks $\to$ higher liquidity premium & lower r
- Capital with commitment about constant, recall $\tilde{\delta} = \frac{1-\Phi}{1+r}$

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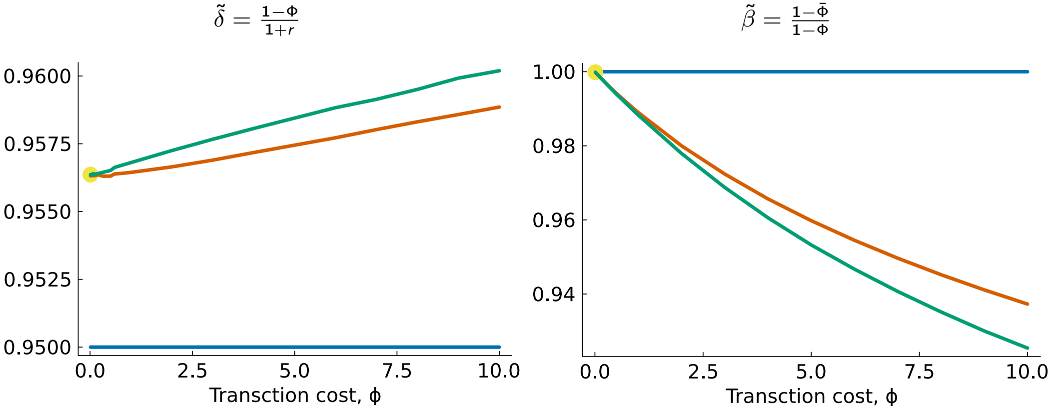
Lack of commitment: Quasi-Hyperbolic Discounting with Present Bias



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Data: Relative Spreads

- Daily data on ordinary shares traded in NYSE (CRSP). Relative spreads:

$$RS_{i,t} = \frac{A_{i,t} - B_{i,t}}{0.5(A_{i,t} + B_{i,t})}$$

- 2000Q1 to 2022Q1 (average of daily data), 3k firms, 124k firm-quarter obs

Relative Spreads, %

	Mean	St. dev.	p10	p50	p90
2000Q1-2022Q1	3.37	2.35	1.54	2.79	5.72
2000Q1-2006Q1	3.23	2.28	1.57	2.77	5.23
2010Q1-2019Q4	2.93	1.71	1.47	2.52	4.8

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Data: Relative Spreads, Weighted by Market Capitalization

Relative Spreads, %

	Mean	St. dev.	p10	p50	p90
2000Q1-2022Q1	2.31	1.26	1.24	1.98	3.78
2000Q1-2006Q1	2.64	1.27	1.39	2.35	4.23
2010Q1-2019Q4	1.88	0.8	1.15	1.69	2.84

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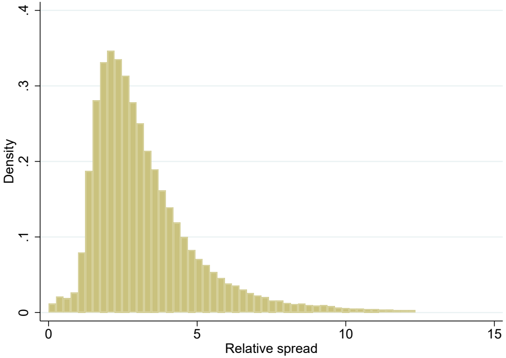
Relative Spreads



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Non targeted moments

	Model	Data
Var log consumption / var log income	0.2	0.3
Mean illiquid assets to GDP	3.7	2.9
Mean liquid assets ($b > 0$) to GDP	0.2	0.6
Share with $b < 0$	0.5	0.2

Consumption and income data from Krueger and Perri (2006). Asset data from SCF 2004 (see Kaplan et al., 2018).

Consistent with non-targeted moments despite being an stylized model without many quantitative add-ons.

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Generalized Weights

$$\int_{\theta,b,h} w(\theta,b,h)\left[d_t + (1-\phi\Delta_t^-(\theta,b,h))q_t\right] d\Gamma_t(\theta,b,h)$$

- Current shareholder weighting (Grossman Hart 1979) is given by $w(\theta,b,h)=\theta$
- Future shareholder weighting (Dreze 1974)) is given by $w(\theta,b,h)=\Theta_{t+1}(\theta,b,h)$
- For the median owner (DeMarzo 1993) consider the median m satisfying

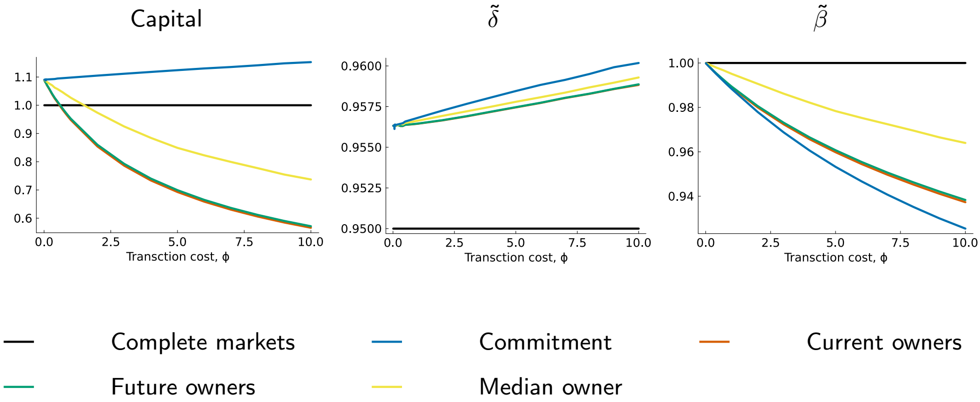
$$\sum_h \int_\theta \int_b 1_{\{\Delta_t^-(\theta,b,h)\leq m\}} \theta d\Gamma_t(\theta,b,h) = \frac{1}{2}$$

and let

$$w(\theta,b,h) = \begin{cases} k \text{ if } \Delta_t^-(\theta,b,h) = m \\ 0 \text{ otherwise} \end{cases}$$

where $k>0$ is chosen so that weights integrate to one.

Generalized Weights



Corporate Bonds

Firms can borrow at interest rate $1 + r^{cb} = \frac{1+r}{1-\tilde{\phi}}$ up to a limit

- If $\tilde{\phi} < \Phi$ the firm always borrows to the limit independently of its commitment.
- If $\Phi < \tilde{\phi} < \overline{\Phi}$ only the firm without commitment borrows up to the limit.

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Implications:

- Can alter financing but not investment and the time-inconsistency problem.
- Firms borrow even if bonds are more illiquid than stocks due to present bias.
- Rationalize corporate debt that does not rely on the tax advantage of debt.

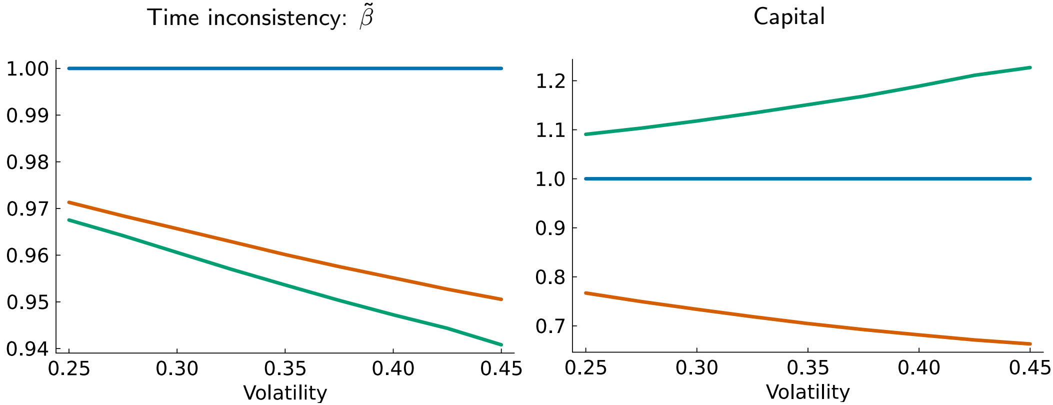
Liquidity & Investment in the Cross-Section

- **Data:** Liquid firms invest more than illiquid ones in the cross-section of US public firms (Amihud and Levi, 2022).
- **Model:** extension with two type of firms, liquid and illiquid ones.

Liquidity & Investment in the Cross-Section

- **Data:** Liquid firms invest more than illiquid ones in the cross-section of US public firms (Amihud and Levi, 2022).
- **Model:** extension with two type of firms, liquid and illiquid ones.
- The liquid firm discounts at rate $\frac{1}{1+r}$ with standard exponential discounting.
- The discount factor of illiquid firms is $\frac{1-\bar{\Phi}}{1+r}$.
- Liquid firms invest more than illiquid ones, consistent with the data.

Demand of Liquidity: Increase idiosyncartic Volatility



- With commitment: more precautionary savings $\rightarrow$ more capital
- Without commitment: more time inconsistency $\rightarrow$ less capital

Liquidity & Investment over the Business Cycle

- **Data:** During recessions markets become less liquid and there is a "flight to liquidity": shift towards more liquid assets (NAES et al. 2011).
- **Model:**
  - Illiquid price-dividend ratio: $q/d = (r + \Phi)^{-1}$
  - Liquid price-dividend ratio: $1/r$
  - Illiquid-to-liquid ratio of the price-dividend ratios: $\left(1 + \frac{\Phi}{r}\right)^{-1}$

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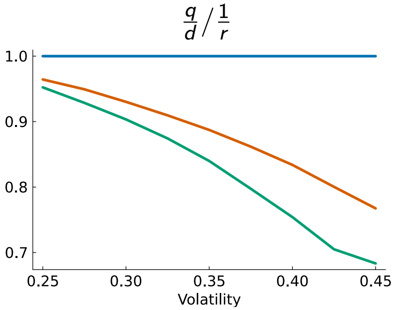
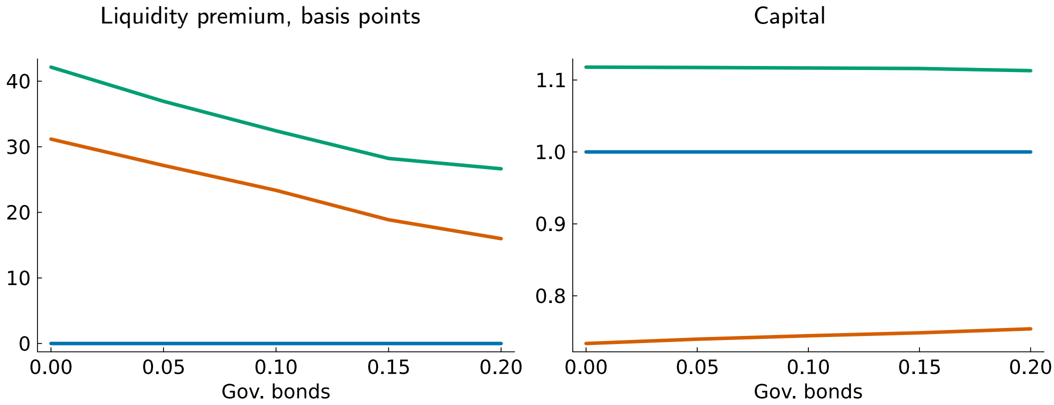


The illiquid-to-liquid ratio of the price-dividend ratios decreases with volatility → *flight to liquidity*

Supply of Liquidity & Government Bonds



Capital closer to complete markets

- With commitment: less precautionary savings $\rightarrow$ less capital
- Without commitment: less time inconsistency $\rightarrow$ more capital

Short-termism

Evidence on short-termism:

- An excessive focus on short-term results at the expense of long-term interests (Graham et al. 2005, Terry 2022, Fink 2015)
- Public firms distort their investment to meet short-term targets (Graham et al., 2005).

Model: Short-termism as a result of (i) trading frictions, and (ii) lack of commitment.

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