Heterogeneous Returns and Portfolio Composition: A Life-Cycle Model Meets U.S. Data

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Wealth Inequality in the U.S.

- Idiosyncratic returns can amplify wealth inequality (e.g., Benhabib et al., 2019).
- Much of the evidence from Norway and Sweden (Bach et al., 2020; Fagereng et al., 2020).
- Quantitative models often abstract from portfolio choice and asset-specific returns.

What do heterogeneous returns look like in the U.S., and what are the implications for wealth inequality?

Data, Model, and Quantitative Results

Data

- Within-asset return heterogeneity is concentrated in private business.
- Business owners are disproportionately at the top of the wealth distribution.
- Portfolios and interactions explain most of the overall return dispersion.

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Quantitative results

- Model reproduces 89% of the wealth Gini, without targeting inequality.
- Equalizing returns lowers the Gini by 32%.
- Inequality depends on who earns high returns and how much they can invest.

▷ Literature

PSID Data on Portfolios and Returns

PSID: Assets and Returns

PSID 1999–2023: 11k households, asset holdings and returns.

Assets:

Liquid

- Fixed income
- Variable income
- Model consolidates into one asset

Illiquid

- Housing equity (net of mortgage)
- Private business equity
- Pensions: IRA and employer DC

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Returns:

- Observe individual income flows for fixed, variable, and business assets.
- Imputed income flows for housing (using mortgage data)
- Pensions: No income flows.
- Capital gains: Imputed for all assets.

▷ Measurement

Four Empirical Facts

1. Within-asset return dispersion is concentrated in private business. ▷ Details
   Variance of 0.2 for business vs less than 0.005 for others (household and year FE)

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3. Portfolios vary strongly with wealth and age. ▷ Wealth ▷ Age
   Business share: 4% for the bottom 80%, 44% for the top 1%.
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4. Variance Decomposition: ▷ Details
   Portfolio composition and interactions explain most return heterogeneity.

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What are the implications for wealth inequality?

A Life-Cycle Portfolio Model with Entrepreneurship

Model Setup

Households

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- Liquid asset: m
- Illiquid assets: business, housing, pensions.
- Endogenous and heterogeneous business income returns
- Other asset returns and all capital-gain rates are common across households.

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Occupation and portfolio choice

- **Portfolio:** At most two illiquid assets; choose pair and levels each period.
- Workers receive labor income and fresh business opportunities.
- Entrepreneurs forgo labor income, face persistent opportunities and forced exit.

▷ Timing ▷ Two-asset restriction ▷ Business opportunities

Endogenous and Heterogeneous Business Returns

Given business equity a, productivity z, and curvature γ:

$$\pi(a, z, \gamma) = \max_{k,b} \; Azk^{\gamma} - r_k \max\{b, 0\}$$

$$k = a + b \geq 0, \qquad b \leq \xi k, \qquad 0 < \gamma < 1$$

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Endogenous return on business equity:

$$d_1(a, z, \gamma) = \frac{\pi(a, z, \gamma)}{a}$$

- Productivity z changes over time.
- Curvature γ stays fixed within a business spell.

Standard static entrepreneurship problem, but equity is business-specific.

▷ Optimal capital

Work, Consumption, and Saving

$$V_j^g(m, \mathbf{a}, \eta, z, \gamma) = \max_{c, \tilde{m}} U(c, a_h) + \beta \mathbb{E}\left[W_{j+1}^g(\tilde{m}, \mathbf{a}, \eta', z', \gamma') \mid \eta, z, \gamma, g \right],$$
$$c + \tilde{m} = m[1 + r_m(m)] + (1 - \tau_y)\left[\phi_g e^{\alpha_j + \eta} + \pi(a_1, z, \gamma)\right] + d_h a_h,$$
$$c > 0, \qquad \tilde{m} \geq \underline{m}$$

Workers: $\phi_2 = 1$

- Labor income and housing cash flows
- No business equity
- New business oppt: (z', γ')

Entrepreneurs: $\phi_1 = \phi_3 = 0$

- Business return includes owner's labor
- Productivity shocks z'
- $\gamma' = \gamma$

▷ Taxes and borrowing ▷ Terminal wealth

Portfolio Choice and Illiquidity

Extensive margin

- Choose business–housing, housing–pensions, or business–pensions.
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Intensive margin

- Reallocate wealth across assets
- Illiquidity: Business and housing sales recover a fraction τ_i of their value.

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Pension rules:

- Model four features: (i) tax deferral, (ii) employer matching, (iii) contribution limits, and (vi) early-withdrawal penalties.

▷ Extensive margin ▷ Intensive margin ▷ Pension rules

Calibration

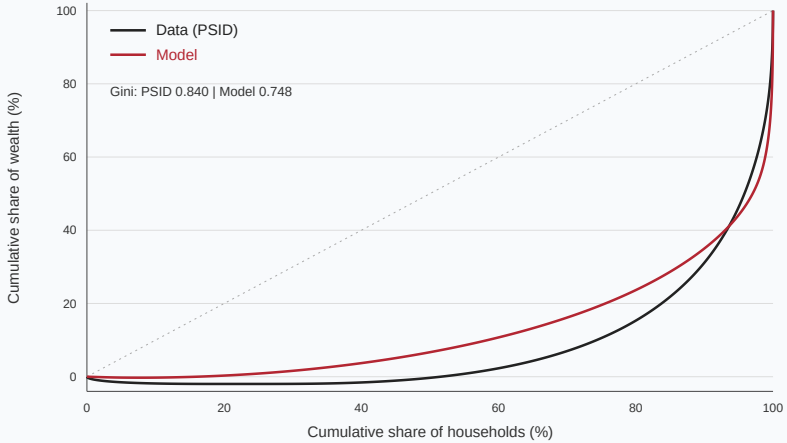
Calibration

- Externally set or estimated
  - PSID labor-income process and initial wealth by income.
  - Asset returns, tax rates, pension rules, transaction costs.
- Thirteen parameters jointly estimated using PSID moments
  - Total wealth: median over life cycle, debt, and housing share.
  - Business wealth: ownership, exit, size distribution, and leverage.
  - Business income-return: level, dispersion, and persistence.

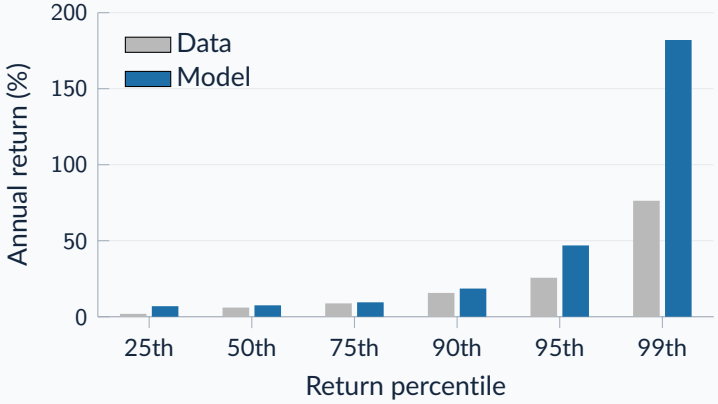
▷ Functional forms ▷ Exogenous parameters ▷ Estimated parameters ▷ Targets

Wealth Inequality and Heterogeneous Returns

The Model Reproduces 89% of the Wealth Gini



Portfolio Returns: A Narrower Middle and a Heavier Upper Tail



- More concentrated in the middle.
- Heavier upper tail.
- Work in progress!

The Mechanisms Behind Wealth Inequality

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Counterfactuals to assess how return heterogeneity shape inequality.

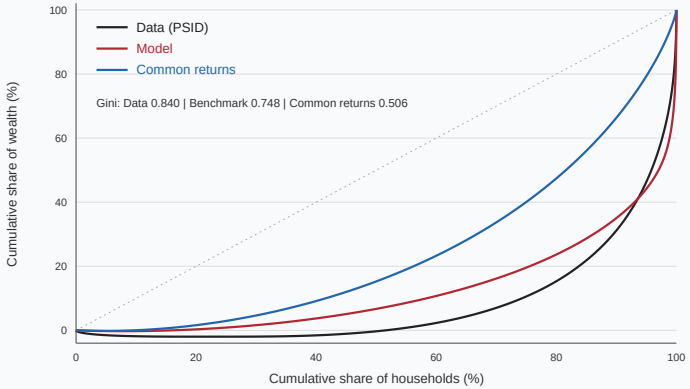
1. Common returns: Equalize gross returns across all assets.
2. Common technology: Fix business productivity z and curvature γ. ▷ Details
3. Scalable business returns: Linear income (heterogeneous or common z). ▷ Details
4. No sale costs: Eliminate the costs of selling business and housing. ▷ Details
5. No pension rules: Replace pensions with unrestricted taxable saving. ▷ Details

Common Returns Reduce the Wealth Gini by One-Third

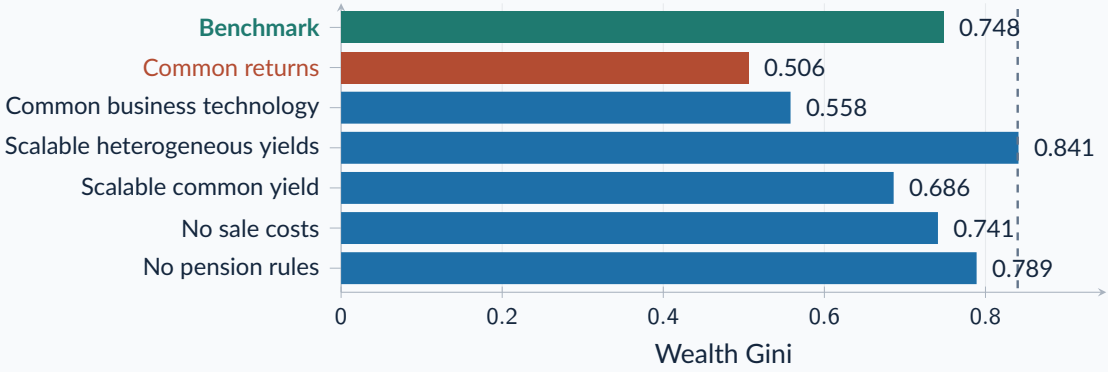
- Give all assets the same return as the mean portfolio of the benchmark economy.
- Replace business production with a constant dividend yield and zero borrowing.

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Wealth Inequality Across Counterfactual Economies



Dashed line: PSID Gini (0.840).

▷ Portfolios ▷ Returns

Conclusion

U.S. evidence

- Private business combines dispersed returns with concentrated ownership.
- Portfolios account for most income-return dispersion among wealthy households.

Quantitative implications

- The model reproduces 89% of the PSID wealth Gini without targeting it.
- Equalizing gross returns lowers the Gini by about one-third.

Literature Review and Contribution

- **Heterogeneous returns and wealth inequality**
  Benhabib, Bisin, and Luo (2019), Fagereng et al. (2020), Bach, Calvet, and Sodini (2020), Hubmer, Krusell, and Smith (2021), Daminato and Pistaferri (2024).
- **Private business income, risk, and returns**
  Hamilton (2000), Moskowitz and Vissing-Jørgensen (2002), Kartashova (2014), Smith et al. (2019), Bhandari and McGrattan (2021).
- **Portfolio choice and entrepreneurship in heterogeneous-agent models**
  Glover et al. (2020), Elenev and Landvoigt (2023), Boar, Gorea, and Midrigan (2025), Brüggemann and Mahone (2025).
- This paper: connect longitudinal U.S. evidence on asset-level returns and portfolios to a life-cycle model with business opportunities and explicit pension institutions.

▷ Back

Return Measurement and Sample

$$r_{h,i,t} = \frac{y_{h,i,t} + CG_{h,i,t}}{W_{h,i,t} + F_{h,i,t}/2}, \qquad F_{h,i,t} = W_{h,i,t+1} - W_{h,i,t} - y_{h,i,t} - CG_{h,i,t}.$$

Dietz (1968). Two-year holding-period returns, CPI-deflated and annualized.

Asset	Income flows	Imputed capital gains
Fixed income	Reported interest	Zero nominal gains
Variable income	Reported dividends	S&P 500 price return
Housing	2.9% of gross value less mortgage interest	State-year FHFA price index
Business	Owner's labor and business income	Russell 2000 price return
Pensions	Unobserved	Excluded from return facts

- Ages 24–63.
- Business equity above $10,000.

▷ Back

Fact 1: Business Returns Are High and Dispersed

	Median annual return (%)		Total-return variance $\times 100$	
	Total	Income	Raw	Person + year FE
Business	39.2	31.1	72.06	20.34
Housing	6.3	1.6	1.47	0.51
Variable income	6.8	1.7	3.65	0.63
Fixed income	-1.6	0.0	0.07	0.03

▷ Back

Fact 2: Business Ownership Rises Sharply with Wealth

	Wealth share (%)			Business owners (%)
Wealth percentiles	PSID	SCF	WID	PSID
0–20	−2.0	0.0	0	1.8
21–40	0.4	1.0	0	3.1
41–60	3.9	4.0	3	6.3
61–80	13.0	11.0	12	12.1
81–95	31.0	24.0	29	23.4
96–99	25.4	26.0	23	39.6
Top 1%	28.2	35.0	34	65.6
Top 0.1%	9.8	—	17	83.6

▷ Back

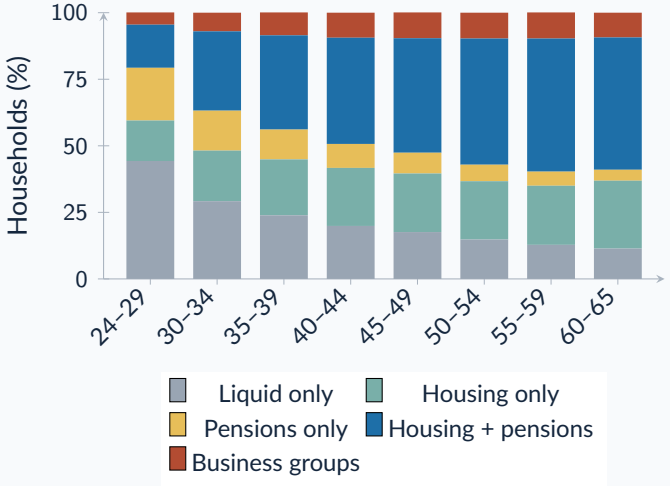
Fact 3: Portfolios Differ Across the Wealth Distribution

	Total Wealth					
Asset	**All**	**Bottom 80%**	**81–90%**	**91–95%**	**96–99%**	**Top 1%**
Business	20.1	3.7	8.0	11.5	18.6	44.3
Housing	36.8	55.2	45.4	38.1	32.5	22.5
Variable Income	13.2	4.4	7.9	11.9	15.4	20.9
Fixed Income	7.1	11.2	8.5	7.6	6.6	3.8
Pensions	22.8	25.5	30.3	30.9	26.8	8.5

- Housing accounts for 55.2% of bottom-80% wealth and 22.5% of top-1% wealth.
- Business accounts for 3.7% of bottom-80% wealth and 44.3% of top-1% wealth.

▷ Two-asset approximation ▷ Back

Fact 3: Portfolios Change over the Life Cycle



- Liquid-only portfolios are most common at ages 24–29.
- Housing plus pensions becomes the modal group from age 30.
- Business groups account for at most about 10% at any age.

▷ Back

Fact 4: Decomposing Income-Return Variance

$$R_i = \sum_a s_{ia} r_{ia}, \qquad \bar{s}_a = \mathbb{E}[s_{ia}], \quad \bar{r}_a = \mathbb{E}[r_{ia}].$$

$$R_i - \sum_a \bar{s}_a \bar{r}_a = \underbrace{\sum_a (s_{ia} - \bar{s}_a)\bar{r}_a}_{P_i\text{: portfolio}} + \underbrace{\sum_a \bar{s}_a (r_{ia} - \bar{r}_a)}_{W_i\text{: within asset}} + \underbrace{\sum_a (s_{ia} - \bar{s}_a)(r_{ia} - \bar{r}_a)}_{I_i\text{: interaction}},$$

$$\mathrm{Var}(R) = \mathrm{Var}(P) + \mathrm{Var}(W) + \mathrm{Var}(I) + \text{covariance terms}.$$

- Shapley–Shorrocks allocates covariance terms across the components.
- Pension returns are unobserved and excluded.

▷ Results ▷ Back

Fact 4: Portfolio Composition Dominates at the Top

	All	Bottom 80%	81–90%	91–95%	96–99%	Top 1%
Share of income-return variance (%)						
Portfolio	51.5	34.9	64.0	66.0	76.0	74.4
Within-asset return	15.0	18.9	9.7	6.6	4.5	6.6
Interaction	33.5	46.2	26.3	27.4	19.6	19.0
Excluding business						
Variance reduction (%)	66.6	43.6	88.1	93.9	95.7	97.7

- Composition alone explains 66–76% of variance above the 90th percentile.
- Removing business reduces variance by 88–98% above the 80th percentile.

▷ Method ▷ Back

State, Occupations, and Within-Period Timing

$$s = (m, \mathbf{a}, \eta, z, \gamma), \qquad I_1 = \{1, 2\},\ I_2 = \{2, 3\},\ I_3 = \{1, 3\}.$$

Asset indices: 1 business, 2 housing, 3 pensions. Group 2 is the worker group.

1. Group choice W_j^g: observe labor income and business opportunities. Incumbents face forced exit. Unrestricted households draw taste shocks and choose ℓ.
2. Reallocation $\Omega_j^{g,\ell}$: capital gains revalue inherited assets. Choose liquid and illiquid holdings, liquidating excluded assets.
3. Consumption and saving V_j^ℓ: earn labor or business income, receive asset cash flows, consume, and choose liquid savings.

- New assets pay cash flows this period and receive capital gains next period.
- At age 24, all households start as workers without illiquid assets and only consume/save. Entry and portfolio reallocation start in period 2.

▷ Back

Why Allow at Most Two Illiquid Assets?

- 94% of household-year observations hold at most two of business, housing, and pensions.
- The asset outside the two largest holdings represents only 2.0% of aggregate illiquid wealth.
- Among business owners, 58% hold all three classes, but the smallest class accounts for only 4.7% of their aggregate illiquid wealth.
- The restriction leaves two continuous reallocation choices within each group.

The approximation retains the dominant portfolios while keeping the dynamic problem tractable.

▷ Back

Business Technology: Optimal Capital

$$k^* = \left(\frac{A\gamma z}{r_k}\right)^{1/(1-\gamma)}, \qquad k^{\text{opt}} = \begin{cases} a, & k^* \le a, \\ k^*, & a < k^* \le a/(1-\xi), \\ a/(1-\xi), & k^* > a/(1-\xi). \end{cases}$$

- Low productivity: operate with own equity.
- Intermediate productivity: choose the unconstrained capital target.
- High productivity: the collateral constraint binds.

▷ Back

Business Opportunities, Entry, and Forced Exit

Incumbent entrepreneurs

- Productivity follows $z' \sim P_z(z, \cdot)$. Curvature stays fixed: $\gamma' = \gamma$.
- Independent forced-exit probability $1 - q_z^{\text{stay}}$.

Workers

- Draw $z' \sim F_z^e$ and $\gamma' \sim F_\gamma$, independently, before deciding on entry.
- F_z^e is the stationary distribution of P_z. Curvature is a business opportunity, not a permanent household trait.

Three distinct reasons for exit

- A forced-exit shock, an unproductive opportunity, or a voluntary occupational switch.
- Zero productivity does not mechanically require exit.

▷ Processes ▷ Choice probabilities ▷ Back

Taxes, Liquid Saving, and Borrowing

$$r_m(m) = \begin{cases} (1-\tau_y)\bar{r}_m, & m \geq 0, \\ \bar{r}_b, & m < 0. \end{cases}$$

- Tax labor income, business cash flows, and positive liquid investment income at τ_y.
- Liquid borrowing interest is not deductible, and $\bar{r}_b > \bar{r}_m$.
- Housing cash flow is untaxed imputed rent.
- Housing holdings are equity net of mortgages.

▷ Back

Terminal Wealth at Retirement

At $T = J + 1$, liquidate assets and value resources for retirement consumption and bequests:

$$w_T^g(s) = m[1 + r_m(m)] + \sum_{i \in I_g} a_i \left[\tau_i^T (1 + \kappa_i) + (1 - \tau_y \mathbf{1}\{i \neq 2\})\, d_i(a_i, z, \gamma) \right],$$

$$(\tau_1^T, \tau_2^T, \tau_3^T) = (\tau_1, \tau_2, 1 - \tau_p),$$

$$W_T^g(s) = \omega_w \frac{[w_T^g(s) + \underline{w}]^{1-\nu_w}}{1 - \nu_w}.$$

- Sale recovery applies to revalued balances. Cash flows are taxed as during working life.
- Pensions face ordinary withdrawal tax, without the working-period penalty.
- The positive shifter bounds marginal utility at zero wealth and also stands in for non-asset retirement resources.

▷ Back

Intensive Margin: Portfolio Reallocation

$$\widetilde{a}_i = (1+\kappa_i)a_i,$$
$$\Omega_j^{g,\ell}(s) = \max_{(m',\mathbf{a}')\in\mathcal{A}_j^{g,\ell}(s)} V_j^{\ell}(m',\mathbf{a}',\eta,z,\gamma).$$

Business and housing, $i = 1, 2$:

$$a_i' = \widetilde{a}_i + x_i - \delta_i, \quad q_i = x_i - \tau_i\delta_i, \quad x_i \geq 0, \quad 0 \leq \delta_i \leq \widetilde{a}_i, \quad x_i\delta_i = 0.$$

Pensions:

$$a_3' = \widetilde{a}_3 + e + M_g(e) - \delta_3, \qquad q_3 = (1-\tau_y)e - R_j\delta_3.$$

Cash balance and portfolio restrictions:

$$m' = m + q_m \geq \underline{m}, \qquad q_m + \sum_i q_i = 0, \qquad a_i' \geq 0, \quad a_i' = 0 \text{ if } i \notin I_\ell.$$

τ_i is a sale recovery fraction. A lower value means a larger cost. Assets excluded from the chosen group must be fully liquidated.

▷ Cash-allocation maps ▷ Back

Extensive Margin, Taste Shocks, and Forced Exit

Type-I extreme-value taste shocks have scale σ_ε and expected maximum zero:

$$\overline{W}_j^g(s) = \sigma_\varepsilon \log\left[\frac{1}{3}\sum_{\ell=1}^{3}\exp\left(\frac{\Omega_j^{g,\ell}(s)}{\sigma_\varepsilon}\right)\right],$$

$$\bar{p}_j^{g,\ell}(s) = \frac{\exp(\Omega_j^{g,\ell}(s)/\sigma_\varepsilon)}{\sum_{r=1}^{3}\exp(\Omega_j^{g,r}(s)/\sigma_\varepsilon)}.$$

For $h_g = (1 - q_z^{\text{stay}})\mathbf{1}\{g \in \{1,3\}\}$:

$$W_j^g(s) = (1-h_g)\overline{W}_j^g(s) + h_g\Omega_j^{g,2}(s),$$

$$p_j^{g,\ell}(s) = (1-h_g)\bar{p}_j^{g,\ell}(s) + h_g\mathbf{1}\{\ell = 2\}.$$

- Forced exit requires the worker group and business liquidation.
- The forced branch has no taste-shock maximization.

▷ Back

Pension Contributions and Employer Matching

A household contribution e costs $(1 - \tau_y)e$ in liquid wealth. The employer adds

$$M_g(e) = \begin{cases} \min\{\mu_p e, \bar{\mu}_p y_j(\eta)\}, & g = 2, \\ 0, & g \in \{1, 3\}. \end{cases}$$

- Workers receive a 50% match, capped at 3% of labor income.
- Entrepreneurs receive the contribution deduction but no employer match.
- Balances compound untaxed. The full withdrawal is taxed, including returns.

▷ Contribution limits ▷ Withdrawals ▷ Back

Pension Contribution Limits

$$Y_j^g(s) = \begin{cases} y_j(\eta), & g = 2, \\ \pi(a_1, z, \gamma), & g \in \{1, 3\}, \end{cases}$$

$$\overline{D}_j^g(s) = \begin{cases} \min\{C_j, Y_j^g(s)\}, & g = 2, \\ \min\{\lambda_p C_j, \theta_p Y_j^g(s)\}, & g \in \{1, 3\}, \end{cases}$$

$$C_j = C_0 + C_{50}\mathbf{1}\{50 \le t_j < 60\} + C_{60}\mathbf{1}\{t_j \ge 60\},$$

$$0 \le e \le \overline{D}_j^g(s).$$

Calibration: $(C_0, C_{50}, C_{60}) = (0.56, 0.18, 0.26)$, $\lambda_p = 2.94$, $\theta_p = 0.25$.

▷ Back

Pension Withdrawals and the Value of Tax Deferral

$$R_j = 1 - \tau_p - \tau_e = 0.65 \quad (j \le J), \qquad R_T = 1 - \tau_p = 0.75.$$

- A withdrawal δ_3 releases $R_j \delta_3$ in liquid wealth.
- $0 \le \delta_3 \le \widetilde{a}_3$ and $e\delta_3 = 0$: no simultaneous contribution and withdrawal.
- The model applies the early penalty throughout working life, then removes it at retirement.

For a contribution held n periods to retirement, with no match:

$$\underbrace{\frac{1-\tau_p}{1-\tau_y}(1+\kappa_3)^n}_{\text{Pension return per unit of liquid resources}} \quad \text{versus} \quad \underbrace{[1+(1-\tau_y)\bar{r}_m]^n}_{\text{Liquid saving}}.$$

With $\tau_p = \tau_y$, the tax advantage is untaxed accumulation. Matching adds to this advantage.

▷ Back

Feasible Portfolios in Net Cash Allocations

Define liquidation values $S_i = \tau_i \widetilde{a}_i$ for $i = 1, 2$, $S_3 = R_j \widetilde{a}_3$, and purchase bounds $U_1 = U_2 = \infty$, $U_3 = (1 - \tau_y)\overline{D}_j^g(s)$.

$$a_i'(q_i) = \begin{cases} \widetilde{a}_i + q_i, & q_i \geq 0, \\ \widetilde{a}_i + q_i/\tau_i, & q_i < 0, \end{cases} \qquad i = 1, 2,$$

$$a_3'(q_3) = \begin{cases} \widetilde{a}_3 + \dfrac{q_3}{1 - \tau_y} + M_g\left(\dfrac{q_3}{1 - \tau_y}\right), & q_3 \geq 0, \\ \widetilde{a}_3 + q_3/R_j, & q_3 < 0. \end{cases}$$

- Retained asset: $-S_i \leq q_i \leq U_i$. New asset: $0 \leq q_i \leq U_i$.
- Exiting asset: $q_i = -S_i$. Absent from both groups: $q_i = 0$.
- Cash balance determines the remaining allocation after two continuous choices.

▷ Exact bounds ▷ Back

Two Continuous Choices: Exact Feasible Bounds

Same group: $I_g = I_\ell = \{r, t\}$

$$\begin{aligned} \underline{m} - m \le q_m \le S_r + S_t, \\ \max\{-S_t, -q_m - U_r\} \le q_t \le \min\{U_t, S_r - q_m\}, \\ q_r = -q_m - q_t. \end{aligned}$$

Change group: $I_g = \{r, s_0\}$ and $I_\ell = \{r, n\}$

$$\begin{aligned} \underline{m} - m \le q_m \le S_r + S_{s_0}, \\ \max\{0, S_{s_0} - q_m - U_r\} \le q_n \le \min\{U_n, S_{s_0} + S_r - q_m\}, \\ q_r = S_{s_0} - q_m - q_n, \qquad q_{s_0} = -S_{s_0}. \end{aligned}$$

- Only q_m with a nonempty second-choice interval are admissible.
- Pension caps restrict feasible purchases. Matching changes the cash-to-balance map.

With no pension advantages or caps, all assets use the ordinary linear purchase/sale map and $U_i = \infty$.

▷ Back

Functional Forms: Preferences and Life Cycle

$$U(c, a_h) = \frac{c^{1-\sigma}}{1-\sigma} \exp(-\gamma_h a_h), \qquad \sigma = 2,$$

$$B(w) = \omega_w \frac{(w + \underline{w})^{1-\nu_w}}{1-\nu_w}, \qquad \nu_w = 2.$$

- Housing services enter through $a_h = a_2$ when held, and zero otherwise.
- 21 biennial working periods, ages 24–65. Retirement node at age 66.

▷ Income and productivity ▷ Back

Functional Forms: Labor Income and Business Opportunities

Labor income

$$y_j(\eta) = \exp(\alpha_j + \eta), \qquad \eta' \sim \Pi_y(\eta, \cdot).$$

α_j is a PSID quartic age profile, normalized to zero at age 24.

Business productivity

$$x' = (1 - \rho_z)\mu_z + \rho_z x + \varepsilon_z', \quad z = e^x,$$
$$\varepsilon_z' \sim N(0, \sigma_z^2), \quad \sigma_z = s_z\sqrt{1 - \rho_z^2}, \quad \mu_z = -s_z^2/2.$$

- Entrants draw from the stationary distribution.
- Curvature follows a Beta distribution with mean $\bar{\gamma}$, SD s_γ.

Labor-income innovations and business opportunities are conditionally independent. Workers redraw productivity and curvature independently.

▷ Back

Exogenous Parameters: Households and Labor Income

Parameter	Description	Value
J	Biennial working periods	21
σ, ν_w	Consumption and terminal-wealth curvature	2
y_ρ	Biennial log-income persistence	0.9212
y_σ	Biennial innovation SD	0.2770
$\sigma_{y,0}$	Initial log-income SD	0.4455
ϕ_2	Worker labor-income fraction	1
ϕ_1, ϕ_3	Entrepreneur labor-income fraction	0

- Income moments and the age profile use the PSID.
- Initial liquid wealth is normal conditional on initial income. All households start without illiquid assets.

▷ Asset returns ▷ Pension parameters ▷ Initial wealth ▷ Back

Exogenous Parameters: Returns and Sale Recovery

Parameter	Description	Value	Source/assumption
$\bar{r}_m$	Liquid return	3.85%	PSID fixed/variable-income mix
r_k	Business borrowing	4.85%	Liquid return + 1 pp
$\bar{r}_b$	Household borrowing	15.85%	Liquid return + 12 pp
d_2	Housing cash flow	1.6%	PSID imputed rent
d_3	Pension cash flow	0	Reinvested within account
κ_1	Business capital gains	3.4%	PSID/Russell 2000 imputation
κ_2	Housing capital gains	5.3%	PSID/FHFA imputation
κ_3	Pension capital gains	6.8%	Variable-income median return
τ_1	Business sale recovery	0.58	Bhandari–McGrattan (2021)
τ_2	Housing sale recovery	0.94	6% selling cost

All returns are annual before household taxes. Liquid wealth combines 64.9% variable income and 35.1% fixed income. Pension advantages enter through explicit rules.

▷ Households ▷ Pension parameters ▷ Back

Exogenous Parameters: Taxes and Pension Provisions

Parameter	Description	Value
τ_y, τ_p	Income tax and pension-withdrawal tax	0.25
τ_e	Working-period pension withdrawal penalty	0.10
C_0	Base worker contribution ceiling	0.56
C_{50}	Catch-up addition, ages 50–59	0.18
C_{60}	Catch-up addition, age 60 and above	0.26
μ_p	Worker employer-match rate	0.50
$\bar{\mu}_p$	Maximum match / labor income	0.03
λ_p	Entrepreneur dollar-ceiling multiplier	2.94
θ_p	Entrepreneur contribution cap / profits	0.25

▷ Pension equations ▷ Households ▷ Back

Jointly Estimated Parameters

Parameter	Description	Estimate
$\beta^{1/2}$	Annual discount factor	0.9444
ω_w	Terminal-wealth utility weight	62.73
$\underline{w}$	Terminal-wealth shifter	4.1357
γ_h	Housing preference	0.0006
$\underline{m}$	Liquid borrowing limit	-0.3125
A	Business productivity scale	0.1316
$\bar{\gamma}$	Mean business curvature	0.7213
s_γ	SD of business curvature	0.1446
s_z	Stationary SD of log productivity	1.9405
ρ_z	Business productivity persistence (biennial)	0.9573
q_z^{stay}	Probability of avoiding forced exit (biennial)	0.8417
ξ	Business collateral parameter	0.1431
σ_ε	Portfolio taste-shock scale	1.1365

Preliminary joint SMM estimates using fourteen targets. Borrowing limit and wealth shifter are in model units.

▷ Wealth targets ▷ Business targets ▷ Back

Targeted Moments: Wealth Accumulation and Portfolios

	Data	Model
Median wealth, ages 64–67 ($)	200,735	184,240
Nonpositive wealth, ages 64–67 (%)	7.87	7.05
Mean wealth, ages 38–42 ($)	177,218	192,978
Mean debt ($)	8,497	8,386
Housing share of aggregate wealth (%)	36.77	48.08

- Wealth levels and debt discipline patience, terminal wealth, and borrowing.
- The main portfolio discrepancy is excess housing in the model.

2000 dollars. Mean debt includes households with zero debt. Late-life moments compare the final model decision at age 64 with PSID ages 64–67.

▷ Business targets ▷ Parameters ▷ Back

Targeted Moments: Business Participation and Returns

	Data	Model
Entrepreneur share (%)	10.47	11.22
Two-year exit rate (%)	36.29	43.61
Median business wealth ($)	83,051	96,936
Business wealth SD / median	22.14	28.18
Owners with business wealth below $25,000 (%)	31.25	21.83
Mean leverage, b/k	0.1156	0.1241
Business income returns (Dietz)		
Median annual income return (%)	31.08	47.92
Return SD / median	2.095	1.288
Two-year autocorrelation of log returns	0.745	0.632

Business wealth uses all owners. Dietz returns apply the empirical computability and cleaning restrictions to simulated data. Autocorrelation uses owners with positive returns in both waves. Exit excludes the last model period. Leverage is winsorized at p99.

▷ Wealth targets ▷ Parameters ▷ Back

Initial Wealth by Income

		Raw ($)			Normalized		
Income Group	N	Mean	Median	SD	Mean	Median	SD
Zero Income	114	-1,345	0	43,889	-0.029	0.000	0.932
Q1 (Lowest)	496	8,602	0	98,204	0.183	0.000	2.085
Q2	633	6,441	0	74,699	0.137	0.000	1.586
Q3	875	9,350	60	104,479	0.199	0.001	2.218
Q4	452	15,515	4,444	53,962	0.329	0.094	1.146
Q5 (Highest)	57	59,585	10,853	187,381	1.265	0.230	3.978

- Each household's first PSID observation between ages 20 and 24.
- Use income-conditional normal draws for liquid wealth at model entry.
- Households start as workers without illiquid assets.

▷ Back

Common Business Technology Reduces Wealth Concentration

- Fix business productivity z and curvature γ at their means.
- Retain endogenous investment and borrowing; realized returns can still vary with equity and financing choices.

	Benchmark	Common technology
Wealth Gini	0.748	0.558
Top-10% wealth share (%)	64.96	36.84
Business share of net wealth (%)	17.43	0.15
Portfolio-return IQR (pp)	1.66	1.35

Removing differences in business opportunities reallocates wealth toward housing and pensions.

▷ Back

Scalable Heterogeneous Business Returns Raise Inequality

- Replace business production with $\Pi(a, z) = a\,cz$ and set business borrowing to zero.
- Normalize c to the benchmark mean business-income yield; retain productivity heterogeneity and forced exit.

	Benchmark	Scalable heterogeneous yields
Wealth Gini	0.748	0.841
Top-10% wealth share (%)	64.96	78.67
Business share of net wealth (%)	17.43	22.53
Portfolio-return IQR (pp)	1.66	1.62

High-productivity households can expand investment without reducing their operating yield.

▷ Back

A Common High Business Yield Still Generates Inequality

- Replace business production with $\Pi(a, z) = aq_C$, a common dividend yield, and set business borrowing to zero.
- Set q_C to the benchmark mean business-income yield; retain capital gains, sale costs, and forced exit.

	Benchmark	Scalable common yield
Wealth Gini	0.748	0.686
Top-10% wealth share (%)	64.96	54.41
Business share of net wealth (%)	17.43	72.99
Portfolio-return IQR (pp)	1.66	32.79

Business returns are common, but portfolio exposure to those returns differs across households.

▷ Back

Removing Sale Costs Mainly Changes Portfolios

- Remove business and housing sale costs: $\tau_1 = \tau_2 = 1$.
- Households receive the full asset value when selling; retain business technology and pension rules.

	Benchmark	No sale costs
Wealth Gini	0.748	0.741
Top-10% wealth share (%)	64.96	64.50
Business share of net wealth (%)	17.43	26.98
Portfolio-return IQR (pp)	1.66	1.64

The business share of wealth rises, while the wealth Gini changes little.

▷ Back

Removing Pension Rules Raises Wealth Concentration

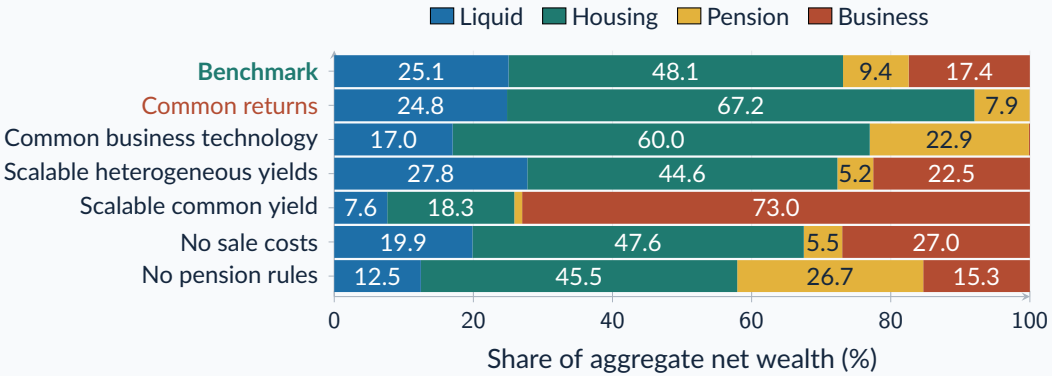
- Replace pensions with unrestricted taxable saving at the same gross return.
- Remove deductions, employer matches, contribution limits, withdrawal taxes, and penalties; tax investment income as ordinary income.

	Benchmark	No pension rules
Wealth Gini	0.748	0.789
Top-10% wealth share (%)	64.96	70.14
Business share of net wealth (%)	17.43	15.33
Portfolio-return IQR (pp)	1.66	1.74

Removing pension rules changes portfolio choices and raises wealth concentration.

▷ Back

Counterfactuals Substantially Reallocate Wealth



- Common returns and common technology make business wealth almost disappear.
- A common high business yield makes business the dominant asset.

▷ Back

Return Dispersion Across Counterfactual Economies



Thick segments: 25th–75th percentiles. Black ticks: medians. Business returns are conditional on ownership.

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