Heterogeneous Returns and Portfolio Composition: A Life-Cycle Model Meets U.S. Data

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Wealth Inequality in the U.S.

- Idiosyncratic returns can amplify wealth inequality (e.g., Benhabib et al., 2019).
- Evidence is mostly Norway/Sweden, not U.S. (Bach et al., 2020; Fagereng et al., 2020).
- Quantitative models often ignore portfolio and asset-specific returns.

What do heterogeneous returns look like in the U.S., and what do they imply for inequality?

Data, Model, and Quantitative Results

Data

- Within-Asset return heterogeneity is concentrated in private business.
- Business owners are disproportionately at the top of the wealth distribution.
- Portfolios and interactions explain most of the overall return dispersion.

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Quantitative results

- Traditional one-asset life-cycle HA model explains about 44 percent of the wealth Gini in the data.
- Portfolios, heterogeneous returns, and liquidity frictions add 22 pp of the wealth Gini.
- About 33 percent of wealth inequality remains unexplained.

▷ Literature

PSID data on portfolios and returns

PSID: Assets and Returns

- Use PSID to track household wealth and portfolios over the life cycle
  - Portfolio data are available from 1997
  - We can link to other household characteristics
- Group assets into five asset classes

Liquid

- Fixed Income (cash, bank accounts, bonds)
- Variable Income (stocks)
- Model: consolidate to one liquid asset

Illiquid

- Housing $\rightarrow$ *private* utility
- Private Businesses $\rightarrow$ entrepreneurship
- Pensions $\rightarrow$ matches and tax benefits

- Net values: assets $-$ debts. Credit-card debt and loans are subtracted from fixed income.

Empirical Facts

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   - The PSID captures the wealth distribution well, similarly to the SCF.
3. Portfolio composition varies strongly by wealth and age. ▷ details
   - Wide dispersion in participation in different asset classes across the wealth and age distribution.
   - Business is more important for wealthier households.
   - Modal portfolio life cycle: Liquid assets $\rightarrow$ Housing $\rightarrow$ Housing + Pensions

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   - Modal portfolio life cycle: Liquid assets $\rightarrow$ Housing $\rightarrow$ Housing + Pensions
4. Variance Decomposition: ▷ details
   - Portfolio composition and interactions explain most return heterogeneity, especially at the top.

Heterogeneous-Agent Macrofinance Model

Model Setup

Households

- Life-cycle HA model with labor risk and warm-glow bequests.

Assets and returns

- Liquid asset: m
- Illiquid assets: business, housing, and pensions.
- Business cash-flow returns are endogenous and heterogeneous.
- Other cash-flows and capital gains are exogenous and constant across households.

Portfolio choice

- At most two illiquid assets; choose pair and levels each period.

Portfolio composition and business productivity determine heterogeneous returns.

▷ Details

Endogenous and Heterogeneous Business Returns

Given business assets a and productivity z:

$$\pi(a, z) = \max_{k,b} zk^{\gamma} - r_k \max\{b, 0\}$$
$$b \leq \xi k \qquad k = a + b$$

Endogenous return on business assets:

$$d_1(a, z) = \frac{\pi(a, z)}{a}$$

Standard static entrepreneurship problem, but assets are business-specific.

▷ Optimal capital

Working-Consumption-Saving

A household of age j, group g, liquid wealth m, illiquid assets (a_1, a_2), labor income y, productivity z solves:

$$V_j^g(m, a_1, a_2, y, z) = \max_{c, \tilde{m}} U\left(c, \overbrace{a_h}^{\text{housing}}\right) + \beta \mathbb{E}\left[W_{j+1}^g(\tilde{m}, a_1, a_2, y', z')\right]$$

$$c + \tilde{m} = \underbrace{\phi_g \exp(\alpha_j + \eta)}_{\text{labor income}} + \underbrace{d_{g_1}(a_1, z) a_1}_{\text{cash flow}} + \underbrace{d_{g_2}(a_2, z) a_2}_{\text{cash flow}} + (1 + r_m)\, m$$

$$\tilde{m} \geq \underline{m} \quad \eta' \sim \Pi_y(\eta, \cdot)$$

Worker ($\phi = 1$)

- Receives labor income
- No active entrepreneurial productivity state

Entrepreneur ($\phi = 0$)

- $d_1(a, z)$ is the ROA of entrepreneurship
- $\log z' = \rho_z \log z + \varepsilon'$ with mixture-normal innovations.

Portfolio Problem

Extensive margin

- Choose asset pair: business-housing, housing-pension, or business-pension.

Intensive margin

- Choose liquid savings and asset levels within the selected pair.

Reallocation frictions

- Capital gains revalue positions.
- Transaction costs make illiquid assets costly to adjust.

▷ Value functions

Taking Stock: Sources of Heterogeneous Returns

- **Portfolios**
  - Liquid, housing, pension, business.
  - Different returns and transaction costs.
- **Private business:** Endogenous and heterogeneous cash-flow returns
  - Entrepreneurial ability: persistent productivity z
  - Entrepreneurial risk: idiosyncratic productivity shocks
  - Illiquidity: transaction costs

Counterfactuals to decompose the role of heterogeneous returns for wealth inequality.

Calibration

Calibration

- Externally set or estimated:
  - Estimate the income process from the PSID.
  - Asset cash-flow returns, capital gains, and transaction costs from aggregate data.

- Internally calibrated with PSID micro moments:
  - Warm glow and debt: final-age wealth moments and average debt.
  - Portfolio composition: business and housing wealth shares.
  - Entrepreneurial productivity: ROA dispersion and persistence, ownership, and exit

▷ Functional Forms & Parameters ▷ Targeted Moments ▷ Non-targeted Moments

Wealth Inequality and Heterogeneous Returns

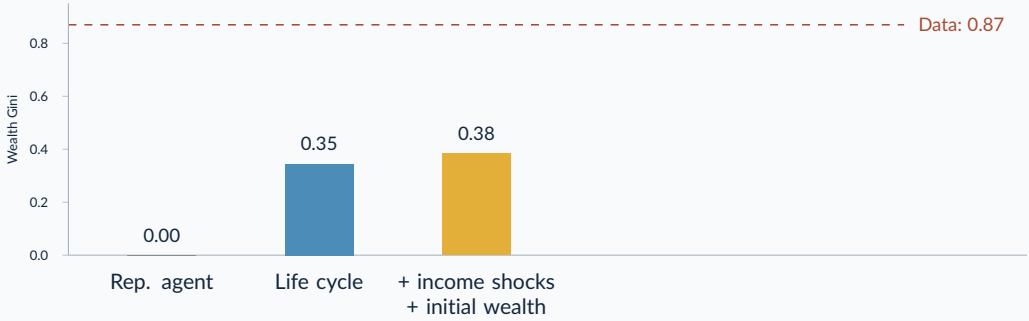
Model Explains 2/3 of Wealth Inequality

	Data	Model
Avg. return IQR (pp)	6.5	4.4
Wealth Gini	0.87	0.58

The model explains $\approx 2/3$ of U.S. wealth inequality and return heterogeneity.

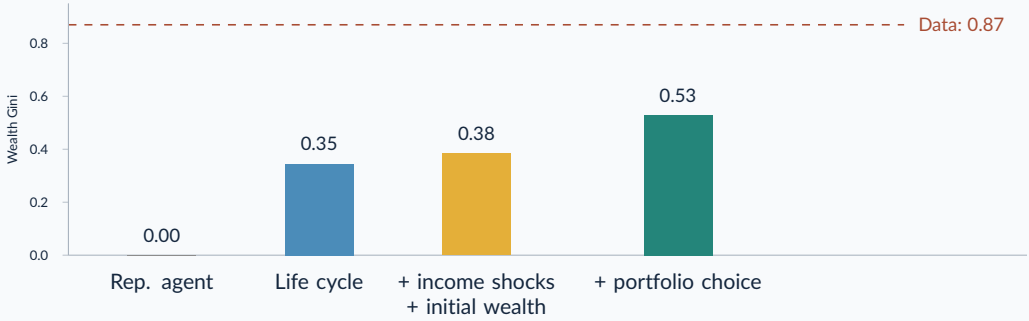
- Return heterogeneity is disciplined by measured returns, not by the wealth Gini.

Decomposition of Wealth Inequality



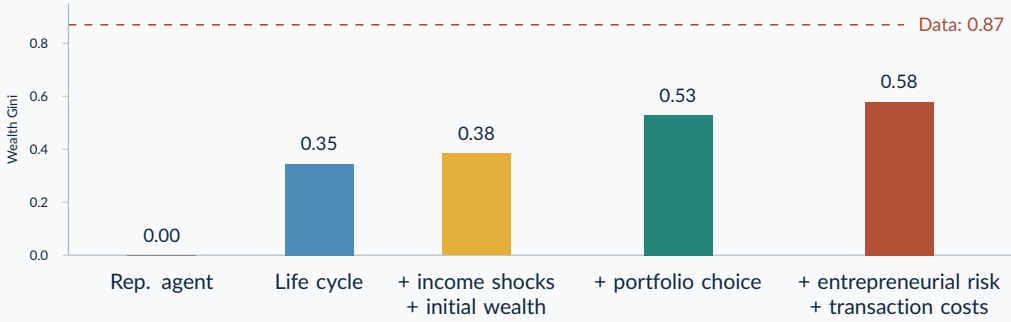
- Traditional one-asset HA generates 44% of data wealth inequality.

Decomposition of Wealth Inequality



- Portfolio choice raises Wealth Gini from 0.38 to 0.53 (17 pp of data).

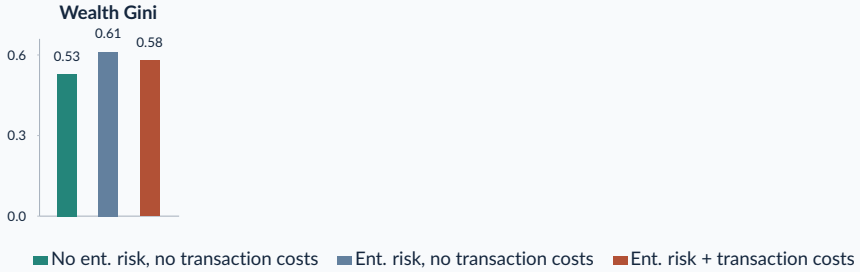
Decomposition of Wealth Inequality



- Full model adds 22 pp beyond the traditional one-asset HA.
- About 33% of wealth inequality remains to be explained by additional forces.

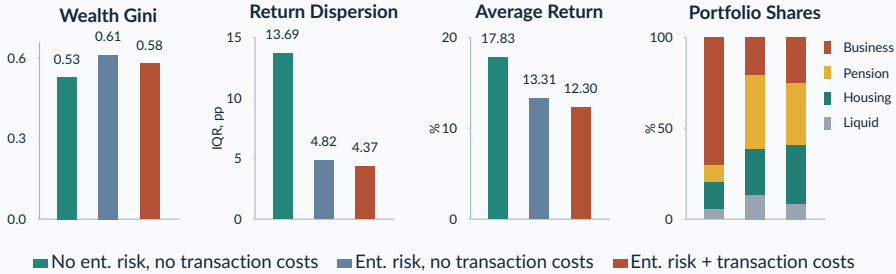
▷ Counterfactual Models

Entrepreneurial Risk: Lower Dispersion, Higher Inequality



▷ Entrepreneurial risk increases wealth inequality.

Entrepreneurial Risk: Lower Dispersion, Higher Inequality



▷ Entrepreneurial risk increases wealth inequality.
▷ Entrepreneurial risk lowers both the mean and dispersion of returns.
▷ Inequality rises because risk reshapes portfolios and saving behavior.

Entrepreneurial risk alone is not sufficient to predict wealth inequality; endogenous portfolios matter.

Conclusion

U.S. PSID data: portfolio composition drives return heterogeneity

- Heterogeneous returns are mainly a portfolio-composition fact.
- Private business is key: dispersed returns, concentrated ownership.

Model implications

- Traditional one-asset HA explains 44% of wealth inequality.
- Portfolios and heterogeneous returns add 22 pp, mostly via portfolio composition.
- Asset specific return dispersion alone misses how risk changes portfolios and saving.

APPENDIX

Literature Review and Contribution

- **Heterogeneous returns and wealth inequality:** Fagereng et al. (2020), Bach et al. (2020), Daminato and Pistaferri (2024), Benhabib et al. (2019), Hubmer et al. (2021).
- **Private business returns:** Hamilton (2000), Moskowitz and Vissing-Jørgensen (2002), Kartashova (2014), Bhandari and McGrattan (2021).
- **Portfolio-based HA models:** Glover et al. (2020), Elenev and Landvoigt (2023), Boar et al. (2025), Brüggemann and Mahone (2025).
- **This paper:** We use PSID panel data to measure asset-level returns and show return heterogeneity is concentrated in private business, then discipline a life-cycle portfolio model with these facts.

▷ Back

Data Appendix

Fact 1: Returns are highly heterogeneous—but only for private business.

Asset Returns

We compute real annualized returns for each household h, asset i, and period t (Dietz, 1968):

$$r_{h,i,t} = \frac{y_{h,i,t}}{W_{h,i,t} + F_{h,i,t}/2}$$

$$r_{h,i,t} = \underbrace{\frac{d_{h,i,t}}{W_{h,i,t} + F_{h,i,t}/2}}_{\text{Income flow}} + \underbrace{\frac{CG_{h,i,t}}{W_{h,i,t} + F_{h,i,t}/2}}_{\text{Capital gains}}$$

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Dietz 68 Estimation of Returns

Define the return for asset i as

$$r_{i,t} = \frac{y_{i,t}}{w_{i,t} + F_{i,t}/2}$$

- Numerator $y_{i,t}$: income from financial assets accrued in period t
- Denominator follows Dietz (1968), sum of beginning-of-period stock of assets $w_{i,t}$ and net flow of gross wealth during the year, F_{it}
- $w_{i,t+1} = w_{i,t} + y_{i,t} + F_{i,t}$

Similar to Fagereng et al. (2020); Kartashova (2014); Moskowitz and Vissing-Jørgensen (2002)

▷ Back

Returns Data

	Income Flows (PSID)	**Capital Gains** (external data)
Fixed Income	Interest income	Inflation erosion
Variable Income	Dividends	Real S&P 500
Housing	Imputed rent (6%) minus mortgage interest	State-year real house price index
Private Business	Self-reported business and labor income	Real Russell 2000
Pensions		Real S&P 500 + 3.5% tax advantage

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Median Returns by Asset Class

	Total	Income	Cap Gains
Business	37.1	26.5	4.3
Housing	6.0	1.6	4.7
Variable Income	7.5	1.8	4.5
Fixed Income	-1.4	0.0	-2.0

- Business has the highest returns
- Business returns are primarily driven by income flows
- Non-business assets have lower typical returns

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Returns Heterogeneity: Variance of Total Returns

	Raw	Inv. Mills	Year FE	Person + Year FE
Business	69.61	69.60	59.95	22.26
Housing	1.19	1.16	0.80	0.47
Variable Income	3.73	3.39	1.27	0.66
Fixed Income	0.04	–	0.04	0.02

- Business returns are an order of magnitude more heterogeneous than other classes.
- Selection: Heterogeneity not driven by selection.
- Aggregate shocks: Year FE reduce variance for all assets, especially variable income.
- Within-household variation: Person FE reduce variance for all assets, especially business.

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Heckman Selection Correction: Setup

Participation equation for each asset class a:

$$D_{i,a,t} = 1\{Z'_{i,a,t}\gamma_a + u_{i,a,t} > 0\}$$

where $D_{i,a,t} = 1$ if household i holds positive wealth in asset a.

Common controls in all first-stage probits: age group $\times$ marital status, year FE, education FE, state FE, parental education.

Asset-specific exclusion restrictions:
- **Business:** father self-employed businessman, log inheritance.
- **Housing:** parental homeownership, number of siblings.
- **Variable income:** log inheritance.

Inverse Mills ratio from these probits is included in return regressions.

▷ Back

First-Stage Power: Participation Equations

	N	Pseudo R^2	Model χ^2	Excl. Restr. χ^2	Excl. p-value
Business	56,596	0.079	2618.8	54.45	0.000
Housing	56,596	0.247	14264.8	196.61	0.000
Variable Income	56,596	0.160	7602.2	346.43	0.000

- Exclusion restrictions are jointly significant in all participation equations (p-value = 0.000).
- First-stage fit is meaningful: pseudo R^2 = 0.08 (business), 0.25 (housing), 0.16 (variable income).

▷ Back

Exclusion Restriction Validity Tests

Asset Class	N	F-statistic	P-value
Business	2,132	2.49	0.083
Housing	23,898	2.73	0.066
Variable Income	4,038	0.07	0.796

Null: exclusions do not predict returns conditional on participation.

- Cannot reject the null for all three assets (all $p > 0.05$).
- Supports exclusion-restriction validity for Heckman correction.
- Fixed income: no selection correction (participation is near-universal, $>$90%).

▷ Back

Variance of Income Flows

	Raw	Inv. Mills	Year FE	Person + Year FE
Business	40.21	40.21	39.03	13.32
Housing	0.15	0.14	0.13	0.07
Variable Income	0.08	0.08	0.08	0.04
Fixed Income	0.02	0.02	0.02	0.01

▷ Back

Variance of Capital Gains

	Raw	Inv. Mills	Year FE	Person + Year FE
Business	18.39	18.37	9.64	4.30
Housing	1.78	1.78	1.37	0.78
Variable Income	5.03	4.94	1.19	0.77
Fixed Income	0.02	0.02	0.01	0.01

▷ Back

Fact 2: Business owners are at the top of the distribution

Business Owners Are at the Top of the Wealth Distribution

Percentiles	PSID	SCF	WID	Bus. Own. (%)
0–20	-1.9	0.0	0	2.3
21–40	0.2	1.0	0	1.8
41–60	2.7	4.0	3	5.8
61–80	10.9	11.0	12	12.5
81–95	31.1	24.0	29	24.9
96–99	28.0	26.0	23	38.8
Top 1%	29.0	35.0	34	55.0
99–99.9	19.5	—	17	52.8
Top 0.1%	9.5	—	17	75.0

- Wealth inequality is captured well in PSID, similar to SCF (De Nardi, 2004)
- Business owners are at the top of the wealth distribution.

▷ Back

Fact 3: Portfolio composition varies strongly by wealth and age

Portfolio Composition Varies Substantially by Wealth

	Total Wealth					
Asset	**All**	**Bottom 80%**	**81–90%**	**91–95%**	**96–99%**	**Top 1%**
Business	19.0	4.2	9.0	11.6	17.6	37.7
Housing	34.4	62.1	44.3	34.7	27.3	21.0
Variable Income	12.6	4.9	8.8	11.4	13.3	18.4
Fixed Income	6.9	13.0	8.5	7.2	5.4	4.0
Pensions	27.2	15.7	29.4	35.0	36.4	18.8

- Business is more important for wealthier households
- Housing is less important for wealthier households

Portfolios matter for wealth distribution: High- and low-wealth households hold different asset classes

▷ Back

Portfolio Simplification

- For each household we focus on the two largest illiquid asset classes
  - All have access to liquid assets (fixed and variable income)
  - Largest two of: Housing, pension, private business
- This is a good approximation and captures more than 90% of illiquid wealth
- This will be helpful for solving the quantitative model
  - In the model, we will only allow for two active types of illiquid assets

▷ Back

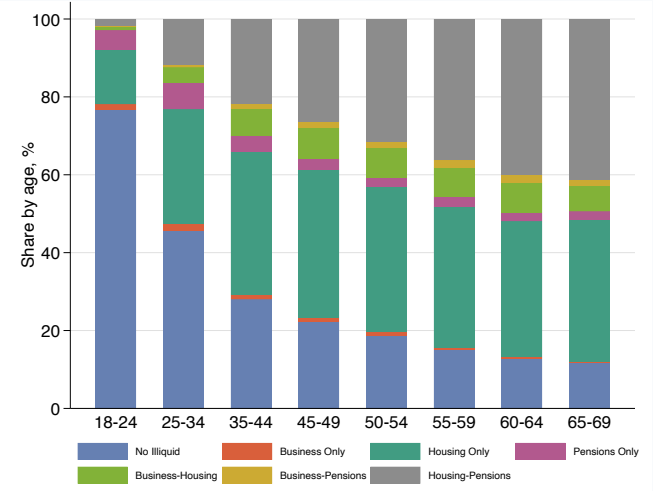
Two Assets Are Enough

Share of (Illiquid) Wealth Covered by Two Main (Illiquid) Assets

	Share (%) of		
Portfolio	**HH Wealth**	**Aggregate Wealth**	**Population**
Housing + Pensions	99.3	54.8	23.0
Business + Housing	96.9	23.8	6.0
Housing Only	100.0	12.4	33.3
Business + Pensions	90.1	7.3	1.3
Pensions Only	100.0	1.8	4.1
Business Only	100.0	0.6	1.1
Liquid Only	100.0	-0.7	31.2

▷ Back

Portfolios Vary over the Life Cycle



- Modal portfolio life cycle:
  1. No illiquid assets
  2. Housing
  3. Housing + Pensions
- The share of business owners remains below 10%

▷ Back

Fact 4: Portfolio Composition Drives Return Heterogeneity

Variance Decomposition of Portfolio Returns

Household portfolio return:

$$R_i = \sum_a s_{ia}\, r_{ia}$$

$$R_i - \bar{R} = \underbrace{\sum_a (s_{ia} - \bar{s}_a)\bar{r}_a}_{P_i\text{: Portfolio effect}} + \underbrace{\sum_a \bar{s}_a (r_{ia} - \bar{r}_a)}_{W_i\text{: Return effect}} + \underbrace{\sum_a (s_{ia} - \bar{s}_a)(r_{ia} - \bar{r}_a)}_{I_i\text{: Interaction}}$$

$$\mathrm{Var}(R) = \mathrm{Var}(P) + \mathrm{Var}(W) + \mathrm{Var}(I) + \text{Cross-Terms}$$

- Portfolio effect (P_i): heterogeneity from differences in asset shares.
- Return effect (W_i): heterogeneity from differences in within-asset returns.
- Interaction (I_i): alignment between portfolio tilts and asset-specific return deviations.
- We use Shapley-Shorrocks decomposition to allocate the cross-terms so the three shares sum to 100%.

▷ Back

Portfolio Composition Drives Return Heterogeneity

	All	Bottom 80%	81–90%	91–95%	96–99%	Top 1%
Variance share						
Portfolio (%)	40.2	32.5	28.4	32.9	37.3	37.0
Return (%)	16.2	16.6	41.2	25.6	18.0	18.7
Interaction (%)	43.5	50.8	30.5	41.5	44.7	44.3
Returns						
Mean (%)	1.4	0.1	5.6	6.7	7.7	9.9
Variance (%)	0.59	0.37	0.83	1.27	1.57	2.39
IQR (%)	6.47	2.78	5.04	5.50	6.04	6.54

- Portfolio composition and interactions account for 84% of return variance.
- Within-asset return heterogeneity accounts for only 16%.

▷ Back

Variance Decomposition Excluding Business

- Remove private business from portfolios

	All	Bottom 80%	81–90%	91–95%	96–99%	Top 1%
Mean return (%)	1.1	0.1	4.9	5.3	5.5	6.6
Variance (%)	0.40	0.33	0.45	0.48	0.49	0.58
Variance reduction (%)	31.7	8.6	45.0	61.9	68.7	75.8

- Reduce variance by 9% for households at the bottom 80%.
- Reduce variance by 76% for households at the top 1%.

Business is the primary driver of return heterogeneity for wealthy households.

▷ Back

Model

Model Setup Details

Life-cycle structure

- Households live for $j = 1, \ldots, J$ periods.
- Idiosyncratic labor income shocks.
- Warm-glow utility over final wealth.

Assets and returns

- Liquid asset m represents cash, bank accounts, and securities.
- Illiquid assets: private business, housing, and pensions.
- Illiquid assets have cash-flow returns d_i and capital gains κ_i.
- Business returns $d_1(a, z)$ are endogenous and heterogeneous.
- Other returns and capital gains are exogenous and constant.

▷ Back

Business Returns: Optimal Capital

Let $k^* = \left(\frac{\gamma z}{r_k}\right)^{1/(1-\gamma)}$. The optimal capital is

$$k = \begin{cases} \frac{a}{1-\xi} & \text{if} \quad k^* \geq \frac{a}{1-\xi} \\ k^* & \text{if} \quad a \leq k^* \leq \frac{a}{1-\xi} \\ a & \text{if} \quad k^* \leq a \end{cases}$$

- Low productivity: operate at business assets a.
- Intermediate productivity: choose unconstrained scale k^*.
- High productivity: collateral constraint binds at $a/(1-\xi)$.

▷ Back

Portfolio Choice and Timing

Two-asset restriction

- Empirically, two asset types characterize the wealth distribution.
- Each household can hold at most two illiquid assets.
- Discrete group choice: $g \in \{(1,2),\ (1,3),\ (2,3)\}$.
- Within group g: choose (a_1, a_2) subject to transaction costs.

Within-period timing

1. Extensive margin: start in asset group g.
2. Intensive margin: choose group / and asset levels.
3. Work/consume/save: entrepreneur problem or labor income.

▷ Back

Portfolio Problem: Intensive Margin

- Household starts in group g with assets m, a_{g_1}, a_{g_2} and moves to group l.
- Choose new portfolio m', a'_{l_1}, a'_{l_2}.

$$\Omega_j^{g,l}\left(m, a_{g_1}, a_{g_2}, y, z\right) = \max_{m', a'_{l_1}, a'_{l_2}} V_j^l\left(m', a'_{l_1}, a'_{l_2}, y, z\right)$$

$$\gamma_m + \sum_{i=1}^{3} \beta_i = \sum_{i=1}^{3} \tau_i \delta_i$$

$$m' = m + \gamma_m$$

$$a'_i = a_i \underbrace{(1 + \kappa_i)}_{\text{Capital gain}} + \beta_i - \delta_i$$

$$\delta_i \in [0, a_i(1 + \kappa_i)] \qquad \beta_i \geq 0 \qquad \gamma_m \geq \underline{m} - m$$

if exit asset i sell all: $\delta_i = a_i(1 + \kappa_i)$

- Cash-flow returns d_i enter the working budget; capital gains κ_i revalue positions.
- Transaction costs $\tau_i < 1$ capture illiquidity in business, pension, and housing.

▷ Back

Intensive Margin: Feasible Portfolio Set

Let $S_i \equiv \tau_i a_i(1+\kappa_i)$ be sale proceeds after capital gains and transaction costs.
k_1, k_2 are current assets; when changing groups, k_1 is the common asset, k_2 is sold, and l_2 is purchased.
Define the net purchases: $\gamma_i = \beta_i - \tau_i \delta_i$

$$\beta_i = \mathbb{I}\{\gamma_i > 0\}\,\gamma_i \qquad \delta_i = \mathbb{I}\{\gamma_i \leq 0\}\,\frac{-\gamma_i}{\tau_i}$$

Lemma 1

If keeping the same illiquid group:

$$\gamma_m \in [\underline{m} - m, S_{k_1} + S_{k_2}]$$
$$\gamma_{k_2} \in [-S_{k_2}, S_{k_1} - \gamma_m]$$
$$\gamma_{k_1} = -\gamma_m - \gamma_{k_2}$$

If changing group:

$$\gamma_m \in [\underline{m} - m, S_{k_1} + S_{k_2}]$$
$$\gamma_{l_2} \in [0, S_{k_1} + S_{k_2} - \gamma_m]$$
$$\gamma_{k_1} = S_{k_2} - \gamma_m - \gamma_{l_2}$$

▷ Back

Calibration

Functional Forms and Processes

Preferences and terminal wealth

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$
$$U(c, a_h) = u(c)\exp\{-\gamma_h a_h\}$$
$$B(w) = \omega_w \frac{\max\{w, \underline{w}\}^{1-\nu_w}}{1-\nu_w} \qquad \text{end-of-life warm glow}$$

- a_h is the housing asset if held, and 0 otherwise.

Income and entrepreneurial productivity

$$y_j = \exp(\alpha_j + \eta), \qquad \eta' \sim \Pi_y(\eta, \cdot)$$
$$\log z' = \rho_z \log z + \varepsilon'$$
$$\varepsilon' \sim p_z N(0, \sigma_z^2) + (1-p_z) N(0, (\lambda_z \sigma_z)^2)$$

▷ Back

Targeted Moments

Parameter	Value	Description	Target moment	Model	Data
Warm glow and Debt					
ω_w	0.41	Warm-glow scale	Final-age median wealth ($000s)	183.62	214.18
$\underline{w}$	0.37	Warm-glow floor	Non-positive wealth share	0.07	0.08
$\underline{m}$	-0.34	Liquid borrowing limit	Mean debt	0.16	0.17
Portfolio Composition					
γ_h	0.02	Housing-service taste	Housing wealth share	0.32	0.34
γ	0.56	Business returns to scale	Business wealth share	0.25	0.19
Entrepreneurial Productivity					
σ_z	0.50	Baseline shock dispersion	ROA IQR	2.01	2.11
p_z	0.82	Baseline-shock probability	Entrepreneurs' exit share	0.34	0.36
λ_z	5.12	High-volatility multiplier	Entrepreneur share	0.10	0.10
ρ_z	0.81	Productivity persistence	ROA autocorrelation	0.64	0.64

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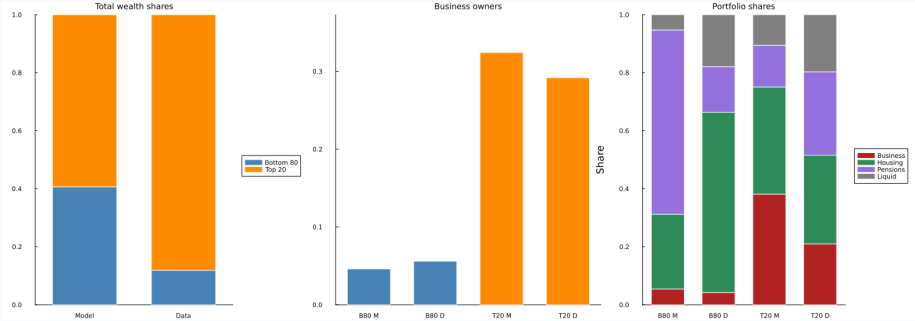
Non-targeted Moments: Portfolios and Wealth



	Data	Model
Median ROA (%)	26.5	22.5

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External Parameters

Parameter	Description	Value	
Preferences			
β	Annual discount factor	0.97	
σ	CRRA parameter	2.00	
ν_w	Terminal-wealth curvature	1.59	
Labor income			
y_ρ	Income persistence	0.92	PSID
y_σ	Income volatility	0.27	PSID
$\sigma_{y,\mathbf{0}}$	Initial-period income volatility	0.45	PSID
Business technology			
ξ	Collateral constraint, $b \leq \xi k$	0.25	PSID leverage
Financial assets (annual rates)			
$\mu_{\text{money}}, \sigma_{\text{money}}$	Initial conditions		PSID
r_k	Entrepreneur borrowing rate	0.055	100 bps spread over liquid
r_m	Return of liquid asset	0.044	PSID, 65% variable income, 35% fixed income
$d_{\mathbf{2}}$	Housing cash-flow return	0.016	Imputed rental rate
$d_{\mathbf{3}}$	Pension cash-flow return	0.000	
$\kappa_{\mathbf{1}}$	Capital gains on business	0.043	Russell 2000
$\kappa_{\mathbf{2}}$	Capital gains on housing	0.047	House price index
$\kappa_{\mathbf{3}}$	Capital gains on pension	0.105	S&P 500 1980-2023 + 3.5% of tax adv.
$\tau_{\mathbf{1}}$	Transaction cost, business	0.58	Bhandari and McGrattan (2020)
$\tau_{\mathbf{2}}$	Transaction cost, housing	0.94	Selling commission
$\tau_{\mathbf{3}}$	Transaction cost, pension	0.65	Penalty + taxes

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Pension Data and Measurement

- PSID pension wealth combines two sources: IRA balances (wealth supplement) and DC accounts (401(k), 403(b), etc.).
- The 2007 PSID redesign introduced bracket responses for DC balances, improving coverage and reducing missing values.
- Including DC accounts substantially raises measured pension wealth (IRA+DC is about 2x IRA-only in both pre-2007 and 2007+ samples).
- Pension income flows are not separately observed in PSID, so household-level pension returns are not directly estimated.

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Initial Portfolios by Income

		Raw ($)			Normalized		
Income Group	N	Mean	Median	SD	Mean	Median	SD
Zero Income	105	-1,896	0	46,675	-0.040	0.000	0.991
Q1 (Lowest)	474	9,394	0	99,530	0.199	0.000	2.113
Q2	606	6,693	0	76,634	0.142	0.000	1.627
Q3	846	8,171	0	102,386	0.173	0.000	2.174
Q4	433	13,181	2,242	54,339	0.280	0.048	1.154
Q5 (Highest)	55	56,883	7,649	191,528	1.208	0.162	4.066

Positive-income groups use model-based cutoffs relative to mean labor income for ages 20–24: Q1 $< 0.366\bar{y}_t$, Q2 $\in [0.366, 0.745)$, Q3 $\in [0.745, 1.519)$, Q4 $\in [1.519, 3.095)$, Q5 $\geq 3.095\bar{y}_t$.

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Pension Return Premium in Calibration

- In the model, pension return is calibrated as equity-like return plus a tax-advantaged premium.
- Discipline this premium using tax deferral, employer matching, fee differentials, and taxable-account tax drag.
- The implied annual premium is about 3 pp on average (larger for shorter horizons because the employer match is front-loaded).
- Pension illiquidity is captured with early-withdrawal costs (10% penalty + income tax), summarized by $\tau_3 \approx 0.65$.

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Wealth Inequality and Heterogeneous Returns

Counterfactual Models

Model	Mechanisms active
Representative agent	One liquid asset; no heterogeneity in wealth.
Only life cycle	One liquid asset with life-cycle saving, no income shocks, and equal initial conditions.
Traditional one-asset HA	One liquid asset with life cycle, income shocks, and different initial conditions.
Portfolio, no risk/no costs	Adds endogenous portfolio choice across liquid assets, housing, business, and pensions.
Portfolio, risk/no costs	Adds entrepreneurial return risk, keeping transaction costs shut down.
Full model	Adds benchmark transaction costs on top of entrepreneurial risk.

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